

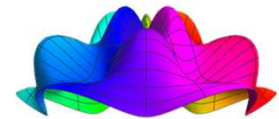
# Spacekime Representation, Statistical Inference & AI prediction using Repeated Measurement Longitudinal Data

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Slides online: Google "SOCR News"

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## Outline

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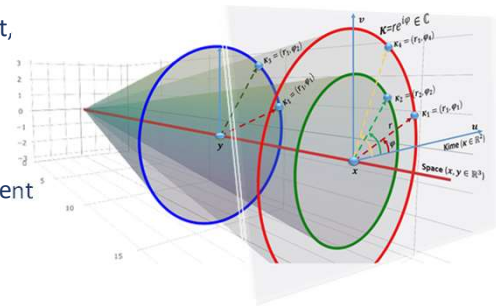
- ☐ *Kime representation of repeated measurement longitudinal processes*
  - ☐ Complex-time (*kime*) & rationale
  - ☐ Kime-phase, random sampling & Heisenberg's Uncertainty
  - ☐ Solutions of ultrahyperbolic wave equations
  - ☐ Mapping Time-series → Kime-surfaces
- ☐ *Kime-Phase Tomography (KPT), recovery of the phase distribution*
- ☐ *Applications: Spacekime Analytics*

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## Complex-Time (Kime)

- ❑ At a given spatial location,  $x$ , complex time (*kime*) is defined by  $\kappa = re^{i\varphi} \in \mathbb{C}$ , where:
  - ❑ The magnitude ( $r > 0$ ) represents the longitudinal order of events characterizing the displacement in time, and
  - ❑ The event phase ( $\varphi \sim \Phi(r)_{[-\pi;\pi]}$ ) is an angular displacement, event direction, reflecting a random sampling index
- ❑ There are multiple alternative parametrizations of kime in the complex plane
- ❑ Space-kime manifold is  $\mathbb{R}^3 \times \mathbb{C}$ :
  - ❑  $(x, k_1)$  and  $(x, k_4)$  are spatially co-localized, but have different kime coordinates,
  - ❑  $(x, k_1)$  and  $(y, k_1)$  are co-localized in kime, but represent different spatial locations,
  - ❑  $(x, k_2)$  and  $(x, k_3)$  have the same spatial-locations and kime-directions, but appear ordered sequentially in time,  $r_2 < r_1$ .

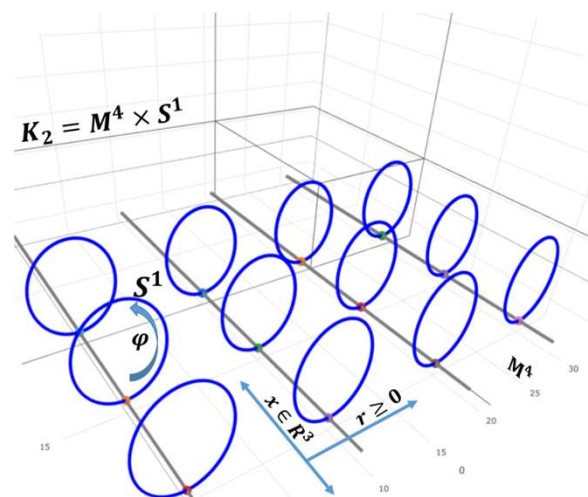


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## Historical Background: Kaluza-Klein Theory

- ❑ Theodor Kaluza (1921) developed a math extension of the classical general relativity theory to 5D. This included the metric, the field equations, the equations of motion, the stress-energy tensor, and the cylinder condition. Physicist Oskar Klein (1926) interpreted Kaluza's 3D+2D theory in quantum mechanical space and proposed that the fifth dimension was curled up and microscopic.
- ❑ The topology of the 5D Kaluza-Klein spacetime is  $K_2 \cong M^4 \times S^1$ , where  $M^4$  is a 4D Minkowski spacetime and  $S^1$  is a circle (non-traversable).



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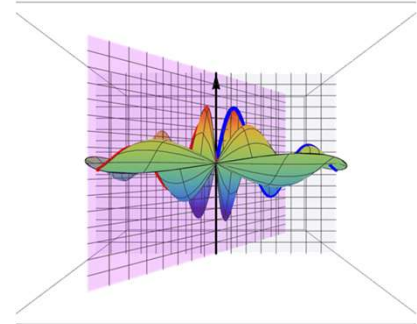
## Rationale for Time $\Rightarrow$ Kime Extension

**Math** – Time is a special case of kime,  $\kappa = |\kappa|e^{i\varphi}$  where  $\varphi = 0$   
**Time** ( $\mathbb{R}^+$ ) is a subgroup of the multiplicative Reals group  
**Whereas kime** ( $\mathbb{C}$ ) is an algebraically closed prime field that naturally extends time  
 Time is ordered but kime is not!  
 Kime ( $\mathbb{C}$ ) represents the smallest natural extension of time, as a complete field that agrees with time

### Physics –

- The Problem of Time: Time has different meanings in *quantum mechanics* & *general relativity*; leading to a tension in formulating a *Quantum Gravity Theory* unifying the two ... (DOI 10.1007/978-3-319-58848-3)
- (Base-field)  $\mathbb{R}$  and  $\mathbb{C}$  Hilbert-space quantum theories make different predictions (DOI: 10.1038/s41586-021-04160-4)

**AI/Data Science** – Random IID sampling, Bayesian reps, tensor modeling of  $\mathbb{C}$  kimesurfaces, novel analytics



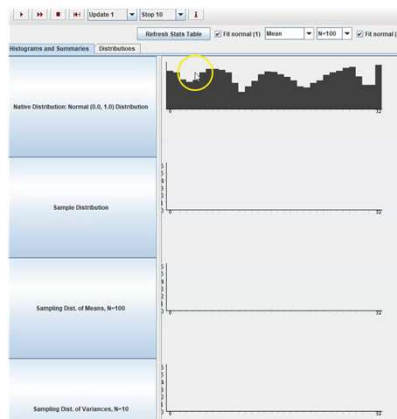
Wesson (2004, 2010)  
 Dinov & Velev (2021)  
 Wang et al. (2022)  
 Zhang et al. (2023)  
 Dinov & Shen (2024)



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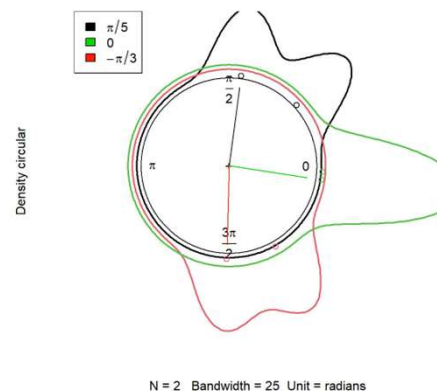
## Random Sampling & Kime-Phase Paradigm

Kime phase distributions are mostly symmetric, random observations  $\equiv$  phase sampling



[https://wiki.socr.umich.edu/index.php/SOCR\\_EduMaterials\\_Activities\\_GeneralCentralLimitTheorem](https://wiki.socr.umich.edu/index.php/SOCR_EduMaterials_Activities_GeneralCentralLimitTheorem)

Kime-Phases Circular distribution



[https://www.socr.umich.edu/TCIU/HTMLs/Chapter6\\_Kime\\_Phases\\_Circular.html](https://www.socr.umich.edu/TCIU/HTMLs/Chapter6_Kime_Phases_Circular.html)



Dinov, Christou & Sanchez (2008)

Dinov & Velev (2021)

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## Kime-Phase Measurement, Observability & Kime Operator

### Kime-Phase Simulation – Repeated Spacetime Measurement

**3 Processes** – Green, Red and Blue colors (scatter points)

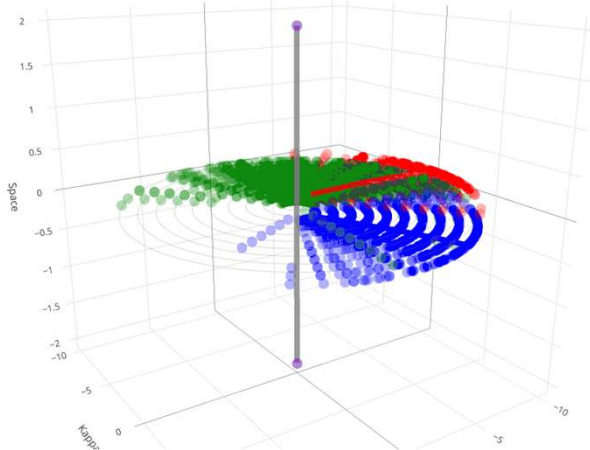
**1 Fixed spatial location** (vertical axis represents 1D space)

**Repeated IID Measurements** colocalized in 4D spacetime

3 Different Kime-Phase distributions (color-coded)

Radial displacement  $t = \text{time}$

Angular (**phase**) location  $\varphi \sim \Phi_{[-\pi, \pi]}(t)$



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Wang et al., 2022 | Dinov & Velez (2021)

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## Ultrahyperbolic PDEs: Wave Equation – Cauchy Initial Data

- For ultrahyperbolic PDEs, the initial value problem, determining the solution(s) for a given initial condition, is **ill-posed**, i.e., there's no guarantee of a global well-defined, stable, and unique solution!
- **Nonlocal constraints** yield the existence, uniqueness & stability of local and global solutions to the ultrahyperbolic wave equation under Cauchy initial data ...

$$\underbrace{\sum_{i=1}^{d_s} \partial_{x_i}^2 u \equiv \Delta_x u(\mathbf{x}, \boldsymbol{\kappa})}_{\text{spatial Laplacian}} = \underbrace{\Delta_{\boldsymbol{\kappa}} u(\mathbf{x}, \boldsymbol{\kappa}) \equiv \sum_{i=1}^{d_t} \partial_{\kappa_i}^2 u}_{\text{temporal Laplacian}}, \quad \begin{cases} u_0 = u\left(\frac{\mathbf{x}}{x \in D_s}, 0, \frac{\boldsymbol{\kappa}_{-1}}{\boldsymbol{\kappa} \in D_t}\right) = f(\mathbf{x}, \boldsymbol{\kappa}_{-1}) \\ u_1 = \partial_{\kappa_1} u(\mathbf{x}, 0, \boldsymbol{\kappa}_{-1}) = g(\mathbf{x}, \boldsymbol{\kappa}_{-1}) \end{cases} \quad \text{initial conditions (Cauchy Data)}$$

where  $\mathbf{x} = (x_1, x_2, \dots, x_{d_s}) \in \mathbb{R}^{d_s}$  and  $\boldsymbol{\kappa} = (\kappa_1, \kappa_2, \dots, \kappa_{d_t}) \in \mathbb{R}^{d_t}$  are the Cartesian coordinates in the  $d_s$  space and  $d_t$  time dims.

Stable local solution over a Fourier frequency region defined by **nonlocal constraints**  $|\xi| \geq |\boldsymbol{\eta}_{-1}|$  :

$$\hat{u}\left(\xi, \frac{\kappa_1, \boldsymbol{\eta}_{-1}}{\boldsymbol{\eta}}\right) = \cos\left(2\pi \kappa_1 \sqrt{|\xi|^2 - |\boldsymbol{\eta}_{-1}|^2}\right) \frac{\hat{u}_0(\xi, \boldsymbol{\eta}_{-1})}{c_1} + \sin\left(2\pi \kappa_1 \sqrt{|\xi|^2 - |\boldsymbol{\eta}_{-1}|^2}\right) \frac{\hat{u}_1(\xi, \boldsymbol{\eta}_{-1})}{2\pi \sqrt{|\xi|^2 - |\boldsymbol{\eta}_{-1}|^2} c_2},$$

$$\text{where } \mathcal{F} \begin{pmatrix} u_0 \\ u_1 \end{pmatrix} = \begin{pmatrix} \hat{u}_0 \\ \hat{u}_1 \end{pmatrix} = \begin{pmatrix} \hat{u}_0(\xi, \boldsymbol{\eta}_{-1}) \\ \hat{u}_1(\xi, \boldsymbol{\eta}_{-1}) \end{pmatrix} = \begin{pmatrix} \hat{u}(\xi, \boldsymbol{\eta}_{-1}) \\ \partial_{\kappa_1} \hat{u}(\xi, \boldsymbol{\eta}_{-1}) \end{pmatrix}.$$

$$u\left(\mathbf{x}, \frac{\kappa_1, \boldsymbol{\kappa}_{-1}}{\boldsymbol{\kappa}}\right) = \mathcal{F}^{-1}(\hat{u})(\mathbf{x}, \boldsymbol{\kappa}) = \int_{\tilde{D}_s \times \tilde{D}_{t-1}} \hat{u}(\xi, \kappa_1, \boldsymbol{\eta}_{-1}) \times e^{2\pi i \langle \mathbf{x}, \xi \rangle} \times e^{2\pi i \langle \boldsymbol{\kappa}_{-1}, \boldsymbol{\eta}_{-1} \rangle} d\xi d\boldsymbol{\eta}_{-1}.$$

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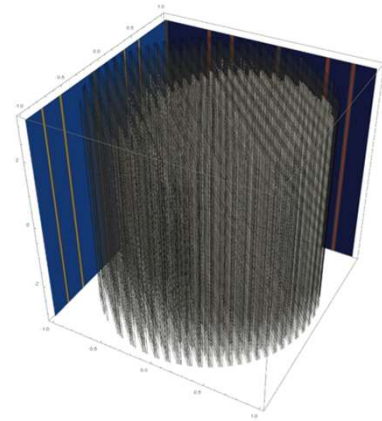
Craig & Weinstein (2008) | Wang et al. (2022) | Dinov & Velez (2021)

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## Ultrahyperbolic Wave Equation – Cauchy Initial Data

### Math Generalizations:

Derived other spacekime concepts: law of addition of velocities, energy-momentum conservation law, stability conditions for particles moving in spacekime, conditions for nonzero rest particle mass, causal structure of spacekime, and solutions of the ultrahyperbolic wave equation under Cauchy initial data ...



(Example Solution in 2D *space* + 2D *kime*)



Wang et al., 2022 | Dinov & Velev (2021)

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## Idea: Longitudinal Data $\Rightarrow$ Kime-Transforms $\Rightarrow$ PDEs $\Rightarrow$ AI

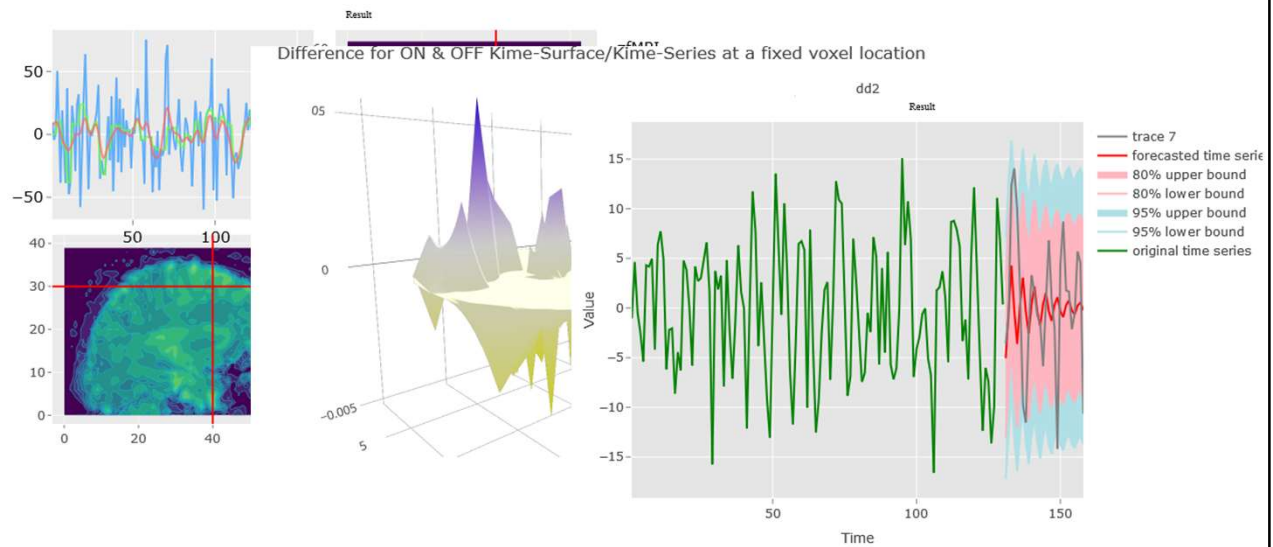
Apps	Time $\rightarrow$ Kime Transformation	Wave equation Solutions (kime) dynamics	Prospective Data Science Applications
Biomed			
	fMRI time-series	fMRI kime-surfaces	Volume rendering
Physics			
	X-ray Diffraction (XRD) Crystallography	XRD Signal	2D Dislocation Strain Field
	Time-dynamic structural phase transitions	Wavelet or Hilbert transform of time-dependent diffraction	Takagi-Taupin PDE model of dynamical X-ray diffraction in deformed crystals
	Bayesian Reconstr. Strain Field	Predict strain fields or defect dynamics	Structural Change Detection (FDR < 0.05)



Rosbach, et al., 2019 | Wang, et al., 2022 | Dinov & Velev (2021)

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## Spacetime Time-series $\Rightarrow$ Spacekime Kimesurfaces $\Rightarrow$ TLM

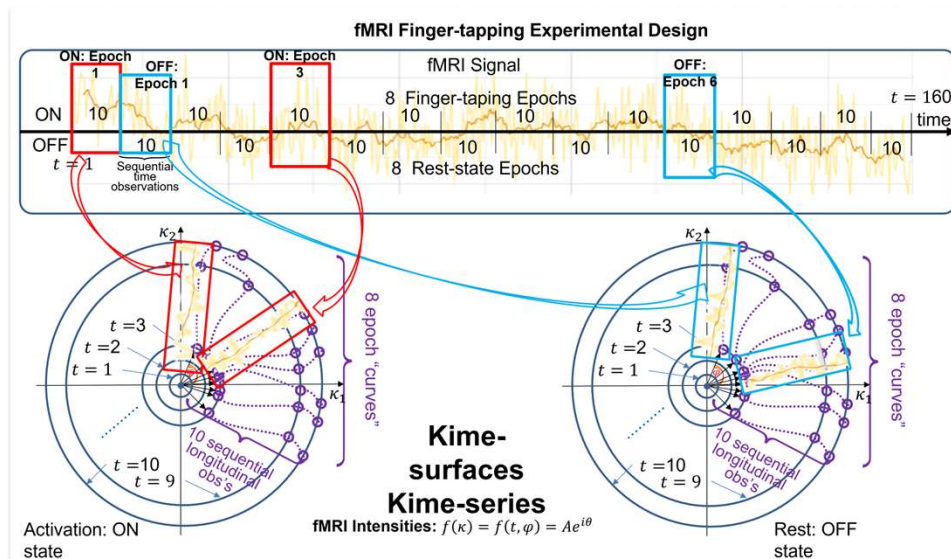


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Zhang et al., 2022 | Dinov &amp; Velez (2021)

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## Mapping Longitudinal Data (Time-series) $\Rightarrow$ Kime-surfaces



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Zhang et al., 2022 | Dinov &amp; Velez (2021)

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## (Analytic) Mapping Time-series $\Rightarrow$ Kime-surfaces

Apply the Inverse Laplace Transform, ILT ( $\mathcal{L}^{-1}$ ) to reconstruct a time-series,  $f(t) = \mathcal{L}^{-1}(F)(t)$ :

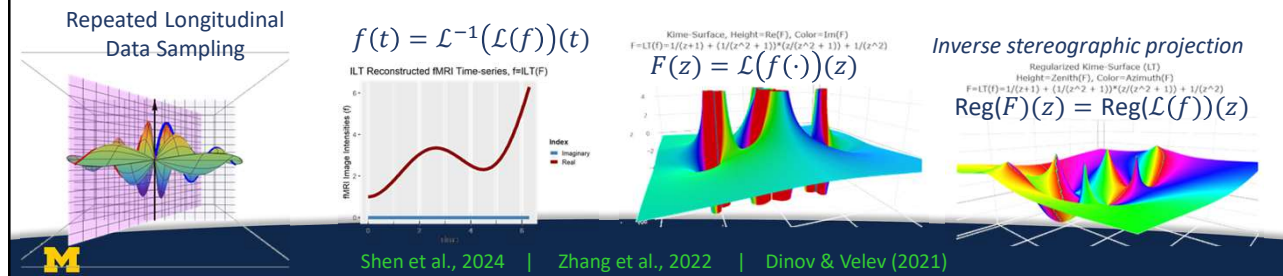
$$F(z) = \mathcal{L}(f) = \frac{1}{z+1} + \frac{1}{z^2+1} \times \frac{z}{z^2+1} + \frac{1}{z^2},$$

$$F_1(z) = \mathcal{L}(f_1(t) = e^{-t}) \quad F_2(z) = \mathcal{L}(f_2(t) = \sin(t)) \quad F_3(z) = \mathcal{L}(f_3(t) = \cos(t)) \quad F_4(z) = \mathcal{L}(f_4(t) = t)$$

$$f(t) = \mathcal{L}^{-1}(F) = \mathcal{L}^{-1}(F_1 + F_2 \times F_3 + F_4) = \mathcal{L}^{-1}(F_1) + \left( \frac{\mathcal{L}^{-1}(F_2) * \mathcal{L}^{-1}(F_3)}{\text{convolution}} \right)(t) + \mathcal{L}^{-1}(F_4) =$$

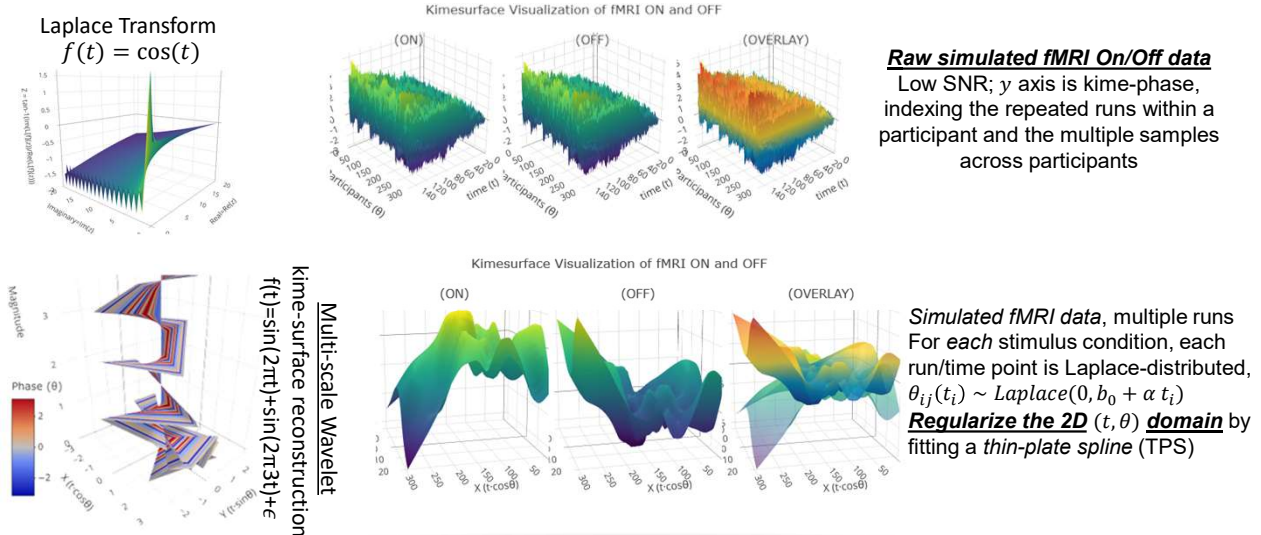
$$\mathcal{L}^{-1}(\mathcal{L}(f_1))(t) + \left( \mathcal{L}^{-1}(\mathcal{L}(f_2)) * \mathcal{L}^{-1}(\mathcal{L}(f_3)) \right)(t) + \mathcal{L}^{-1}(\mathcal{L}(f_4))(t),$$

$$f(t) = \mathcal{L}^{-1}(F)(t) = f_1(t) + (f_2 * f_3)(t) + f_4(t) = e^{-t} + \int_0^t \sin(\tau) \times \cos(t - \tau) d\tau + t = t + e^{-t} + \frac{t \sin(t)}{2}.$$



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## Mapping Longitudinal Data (Time-series) $\Rightarrow$ Kime-surfaces



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[https://www.socr.umich.edu/TCIU/HTMLs/Chapter6\\_TCIU\\_MappingLongitudinalTimeseries\\_2\\_Kimesurfaces.html](https://www.socr.umich.edu/TCIU/HTMLs/Chapter6_TCIU_MappingLongitudinalTimeseries_2_Kimesurfaces.html)

## Kime-Phase Tomography (KPT), phase recovery

- ❑ *Kime representation of repeated measurement longitudinal processes*
  - ❑ Complex-time (*kime*) & rationale
  - ❑ Kime-phase, random sampling & Heisenberg's Uncertainty
  - ❑ Solutions of ultrahyperbolic wave equations
  - ❑ Mapping Time-series → Kime-surfaces
- ❑ *Kime-Phase Tomography (KPT), recovery of the phase distribution*
- ❑ *Spacekime Analytic Applications*



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## Kime-Phase Tomography (KPT), phase recovery

**Definition 1** (*Kime-Domain Signal Space*). Let  $\mathcal{H}_t = L^2(\mathbb{R})$  be the Hilbert space of square-integrable complex-valued functions on the time domain, with inner product  $\langle f, g \rangle_{\mathcal{H}_t} = \int_{\mathbb{R}} f(t) \overline{g(t)} dt$ .

**Definition 2** (*Phase-Domain Space*). Let  $\mathcal{H}_\theta = L^2([-\pi, \pi])$  be the Hilbert space of square-integrable functions on the phase domain, with inner product  $\langle \psi, \phi \rangle_{\mathcal{H}_\theta} = \int_{-\pi}^{\pi} \psi(\theta) \overline{\phi(\theta)} d\theta$  equipped with periodic boundary conditions  $\psi(-\pi) = \psi(\pi)$ .

**Definition 3** (*Kime Space*). The kime space  $\mathcal{K}$  is defined as the tensor product  $\mathcal{H}_t \otimes \mathcal{H}_\theta$ , representing signals in both time and phase domains.

**Definition 4** (*Reproducing Kernel Hilbert Space, RKHS*). The RKHS  $\mathcal{R}_K$  is a subspace of  $\mathcal{H}_t$  with reproducing kernel  $K: \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{C}$  satisfying

- For any  $t \in \mathbb{R}$ ,  $K(\cdot, t) \in \mathcal{R}_K$ , and
- For any  $f \in \mathcal{R}_K$  and  $t \in \mathbb{R}$ ,  $f(t) = \langle f, K(\cdot, t) \rangle_{\mathcal{R}_K}$

**Definition 5** (*Kime-Phase Distribution*). A kime-phase distribution  $\Phi(\theta; t)$  is a time-dependent probability density function on  $[-\pi, \pi]$  satisfying  $\Phi(\theta; t) \geq 0$ ,  $\int_{-\pi}^{\pi} \Phi(\theta; t) d\theta = 1 \quad \forall t \in \mathbb{R}$ .



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## Kime-Phase Tomography (KPT), phase recovery

**Definition 6 (Complex Kime).** For each time  $t$ , the complex kime is defined as  $\kappa(t) = te^{i\theta(t)}$ , where  $\theta(t) \sim \Phi(\cdot; t)$ .

**Definition 7 (Primary Kime Operators).** For a signal  $s(t)$ , the primary kime operators include

1. *Time-domain operator:*  $K_1: \mathcal{H}_t \rightarrow \mathcal{H}_t$  defined by  $K_1[s](t) = t \cdot s(t)$ .
2. *Frequency-Domain Operator:*  $K_2: \mathcal{H}_t \rightarrow \mathcal{H}_t$  defined by  $K_2[s](t) = -i \frac{d}{dt} s(t)$ , and
3. *Scale-Domain Operator:* For a mother wavelet  $\psi \in \mathcal{H}_t$ , let  $W_\psi[s](a, b) = \langle s, \psi_{a,b} \rangle_{\mathcal{H}_t}$  be continuous wavelet transform with  $\psi_{a,b}(t) = \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right)$ . Then, the scale-domain operator  $K_3: \mathcal{H}_t \rightarrow \mathcal{H}_t$  is
 
$$K_3[s](t) = \int_{\mathbb{R}} \int_{\mathbb{R}_+} W_\psi[s](a, b) \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right) \frac{da db}{a^2}.$$
4. *Phase-Domain Operators:* In  $\mathcal{H}_\theta$ , we can define a pair of QM-equivalent phase-domain operators:
  - a. *Position operator:*  $\theta[\phi](\theta) = \theta \cdot \phi(\theta)$ , and
  - b. *Momentum operator:*  $P[\phi](\theta) = -i \frac{d}{d\theta} \phi(\theta)$ .
5. *RKHS Projection Operator:* Given a kernel  $K$ ,  $\mathcal{P}_K: \mathcal{H}_t \rightarrow \mathcal{R}_K$  is defined by  $\mathcal{P}_K[s](t) = \int_{\mathbb{R}} s(\tau) K(t, \tau) d\tau$ .

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## Kime-Phase Tomography (KPT), phase recovery

**Definition 8 (Observable Signal).** An observable kime-signal  $s(t)$  with amplitude  $A(t)$  and phase  $\phi(t)$  is defined as  $s(t) = A(t)e^{i\phi(t)}$ , where  $\phi(t)$  is sampled from distribution  $\Phi_{[-\pi, \pi]}(\cdot; t)$ .

**Definition 9 (fMRI BOLD Signal Model).** In fMRI, the observed BOLD signal  $x(t)$  can be modeled as  $x(t) = \int_{\mathbb{R}} h(t - \tau) s(\tau) d\tau + \epsilon(t)$ , where  $h(t)$  is the *hemodynamic response function* and  $\epsilon(t)$  is *noise*.

**This kime-operator framework is used for kime-phase recovery using repeated measurement observations of a controlled experiment, e.g., repeated fMRI runs in an event-related block design.**

**Theorem 1 (Time-Frequency Commutation).** The operators  $K_1$  (time-domain operator) and  $K_2$  (frequency-domain operator) are *incompatible*, i.e., they have a non-trivial commutator,  $[K_1, K_2] = K_1 K_2 - K_2 K_1 = i\mathcal{I}$ , where  $\mathcal{I}$  is the identity operator on  $\mathcal{H}_t$ . This indicates that the phase-reconstructions corresponding to this pair of kime-operators differentially probe the kime-phase and jointly, they recover complementary phase information.

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## Kime-Phase Tomography (KPT), phase recovery

**Theorem 2 (Uncertainty Relation).** Given a signal  $s \in \mathcal{H}_t$ , time & frequency operators are non-commutative

$$\Delta K_1 \cdot \Delta K_2 \geq \frac{1}{2} |\langle [K_1, K_2] \rangle| = \frac{1}{2},$$

where  $\Delta K_j = \sqrt{\langle K_j^2 \rangle - \langle K_j \rangle^2}$  for  $j = 1, 2$  and expectations are with respect to  $s$ .

**Theorem 3 (RKHS Representation).** Given a signal  $s \in \mathcal{H}_t$  and a reproducing kernel  $K$ , the phase function  $\phi(t)$  can be represented as the *complex argument* of the RKHS projection operator,  $\mathcal{P}_K: \mathcal{H}_t \rightarrow \mathbb{C} \ni \mathcal{P}_K[s](t)$ , i.e.,  $\underbrace{\phi(t) = \text{Arg}(\mathcal{P}_K[s](t))}_{\text{Phase Estimate}}$ .



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## Kime-Phase Tomography (KPT), phase recovery

**Lemma (Phase Recovery from Multiple Bases).** Given repeated observations in multiple non-commuting bases defined by (time, freq & scale) operators  $K_1, K_2, K_3$ , the kime-phase distribution  $\Phi(\theta; t)$  can be *uniquely determined* if the observations are sufficient.

**Theorem 4 (Generalized Phase Recovery from Multiple Bases).** Given a kime-phase distribution  $\Phi(\theta; t)$  and assuming sufficient observations in multiple non-commuting bases defined by operators  $K_1, K_2, K_3$ , the phase distribution can be uniquely determined under certain uniqueness conditions

1. *Trigonometric Moment Identifiability:* A circular distribution is uniquely determined by its complete set of trigonometric moments  $\{\alpha_k, \beta_k\}_{k=1}^{\infty}$  where  $\alpha_k = \mathbb{E}[\cos(k\theta)]$  and  $\beta_k = \mathbb{E}[\sin(k\theta)]$ ,
2. *Information Complementarity:* The (time, freq & scale) operators  $K_1, K_2, K_3$  must provide complementary information about different moments of the phase distribution, and
3. *Sufficiency Condition:* The observations must constrain enough trigonometric moments to uniquely specify  $\Phi(\theta; t)$  within the class of distributions being considered.



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## Algorithm: Kime-Phase Tomography (KPT), *fMRI Sim*

**Input:** BOLD time series  $\{x_n(t)\}_{n=1}^N$  and kernel  $K$

**Output:** Estimated phase  $\hat{\phi}(t)$

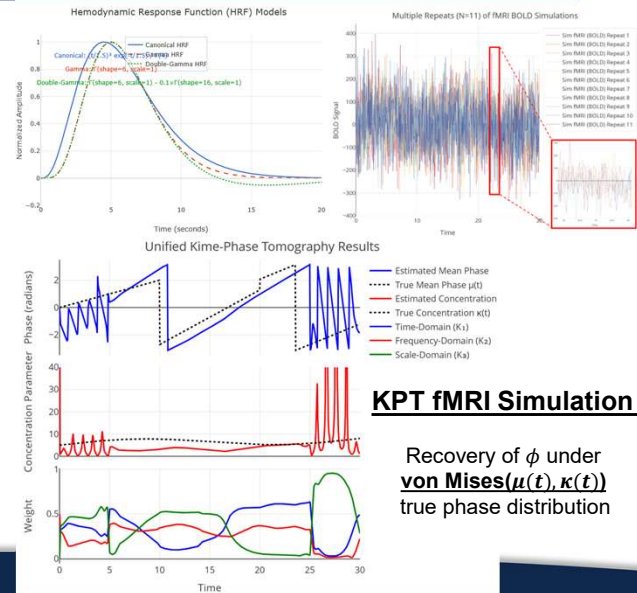
1. Project each signal to RKHS:  $s_n^K(t) = \mathcal{P}_K[s_n](t)$
2. Compute time-domain phase:  

$$\phi_1^K(t) = \arg(t \cdot s_n^K(t))$$
3. Compute frequency-domain phase using STFT,  

$$\phi_2^K(t) = \arg(\mathcal{F}[s_n^K \cdot w](t, \omega_{\max}))$$
, where  $\omega_{\max}$  is the frequency with maximum power
4. Compute scale-domain phase using CWT:  

$$\phi_3^K(t) = \arg(W_\psi[s_n^K](a_{\max}, t))$$
, where  $a_{\max}$  is the scale with maximum power
5. Compute weighted average (*ensemble KPT*):

$$\hat{\phi}(t) = \arg\left(\sum_{j=1}^3 w_j(t) e^{i\phi_j^K(t)}\right).$$



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## Applications: Spacekime Analytics

- ☐ *Kime representation of repeated measurement longitudinal processes*
  - ☐ Complex-time (*kime*) & rationale
  - ☐ Kime-phase, random sampling & Heisenberg's Uncertainty
  - ☐ Solutions of ultrahyperbolic wave equations
  - ☐ Mapping Time-series  $\rightarrow$  Kime-surfaces
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- ☐ *Applications: Spacekime Analytics*

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## Example: Tensor-based Linear Modeling of fMRI

**3-Step Analysis:** registering the fMRI data into a brain atlas space, 56 ROIs, tensor linear modeling, post-hoc FDR processing & selection of large clusters of significant voxels are identified within the important ROIs:

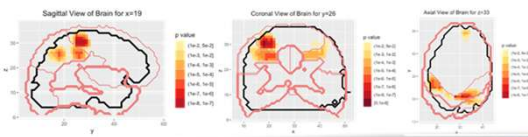
$$Y = \underbrace{\langle X, B \rangle}_{\text{tensor product}} + E$$

time      ROI b-box

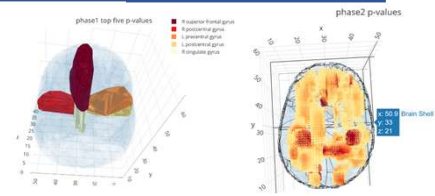
The dimensions of the time-tensor  $Y$  are  $160 \times a \times b \times c$ , where the tensor elements represent the response variable  $Y[t, x, y, z]$ , i.e., fMRI intensity. For fMRI magnitude (real-valued signal), the design kime-tensor  $X$  dimensions are:

$$\text{Kim (Time} \times e^{i \times \text{Repeat}}) \times \text{State} \times \text{effects} \times \frac{1}{\mathbb{R}}.$$

10 \* 8      State      4      1

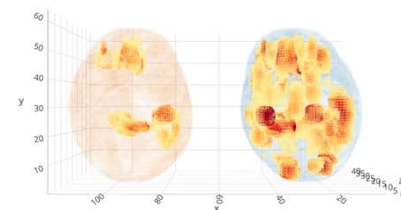


Step 3: 2D voxel analysis projections (finger-tapping task modeling)



Step 1: ROI analysis

Step 2: Voxel analysis



Voxel-based TLM/Analysis  
Corrected (step 3, left) vs. Raw (step 2, right)



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## Spacekime Open-Problems

- ☐ There are many unsolved **abstract mathematical** challenges, e.g., space-kime ergodicity, metric tensor, kime-operator(s), etc.
- ☐ **Numerical & Computational** problems, e.g., reliable kime-phase tomography (KPT), optimal time-series  $\Rightarrow$  kime-surface reconstructions, etc.
- ☐ **Physics** parallels, e.g., contrasting QM vs. Spacekime predictions, physical observability, spacekime measurement, and kime-operator formalism
- ☐ **Analytical** challenges, e.g., new AI techniques for kime-surfaces, analytical verifiability & falsifiability of spacekime theory



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# Spacekime Analytics Tutorial

## TCIU/Spacekime Analytics Tutorial:

### Basic TCIU Protocol for Predictive Spacekime Analytics using Longitudinal Data

Spacekime Analytics (Time Complexity and Inferential Uncertainty)

Basic TCIU Protocol for Predictive Spacekime Analytics using Repeated-Measurement Longitudinal Data

SOCR Team  
10/23/2024

This [Spacekime TCIU Learning Module](#) presents the core elements of spacekime analytics including:

- Import of repeated measurement longitudinal data,
- Numeric (stitching) and analytic (Laplace) kimesurface reconstruction from time-series data,
- Forward prediction modeling extrapolating the process behavior beyond the observed time-span  $[0, T]$ ,
- Group comparison discrimination between cohorts based on the structure and properties of their corresponding kimesurfaces. For instance, statistically quantify the differences between two or more groups,
- Unsupervised clustering and classification of individuals, traits, and other latent characteristics of cases included in the study,
- Construct low-dimensional visual representations of large repeated measurement data across multiple individuals as pooled kimesurfaces (parameterized 2D manifolds),
- Statistical comparison, topological quantification, and analytical inference using kimesurface representations of repeated-measurement longitudinal data.

1 Preliminary setup

TCIU and other R package dependencies ...

[https://www.socr.umich.edu/TCIU/HTMLs/Chapter6\\_TCIU\\_Basic\\_SpacekimePredictiveAnalytics.html](https://www.socr.umich.edu/TCIU/HTMLs/Chapter6_TCIU_Basic_SpacekimePredictiveAnalytics.html)

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## Available Resources

- ☐ SOCR Motto – *“It’s Online & Freely Accessible, Therefore it Exists!”*
- ☐ Pubs: <https://socr.umich.edu/people/dinov/publications.html>
- ☐ GitHub: <https://github.com/SOCR>
- ☐ Datasets: [https://wiki.socr.umich.edu/index.php/SOCR\\_Data](https://wiki.socr.umich.edu/index.php/SOCR_Data)
- ☐ AI Apps: <https://socr.umich.edu/HTML5/> (SOCR AI Bot)
- ☐ Demos: <https://DSPA2.predictive.space>
- ☐ Tutorials: <https://TCIU.predictive.space> & <https://SpaceKime.org>
- ☐ Website: <https://socr.umich.edu>
- ☐ Contact: [statistics@umich.edu](mailto:statistics@umich.edu)

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## Collaborators

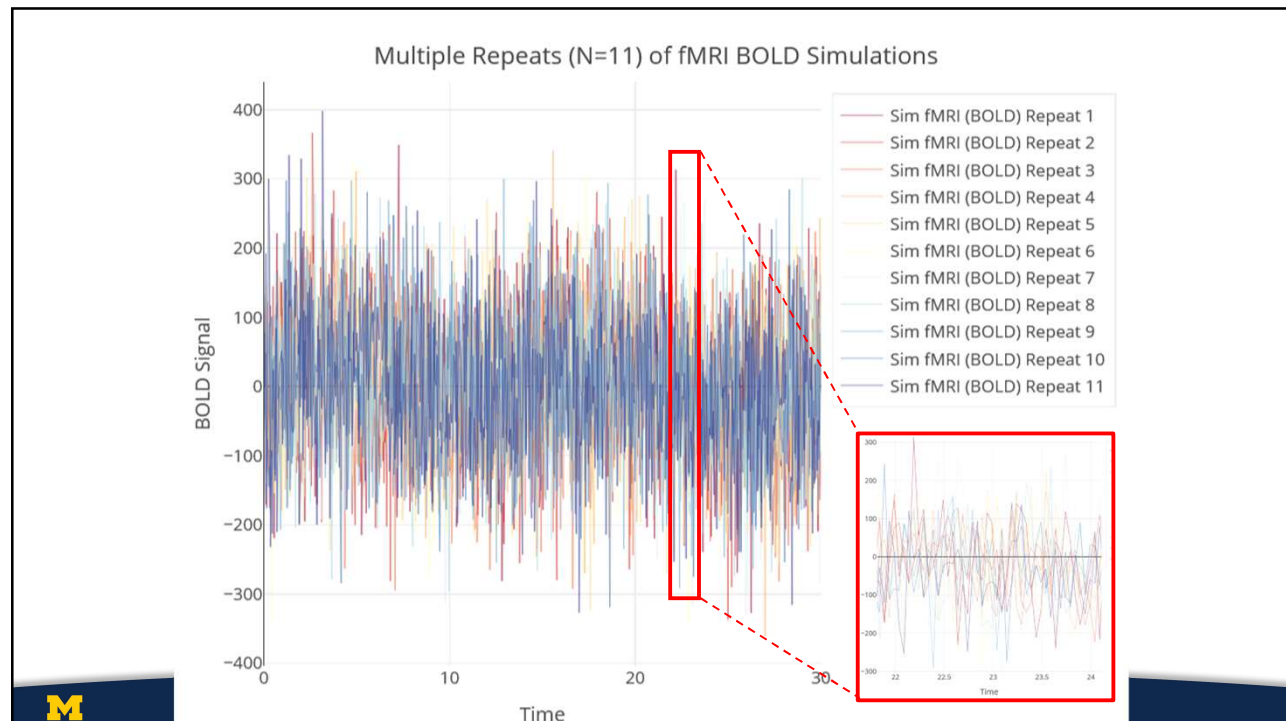
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