





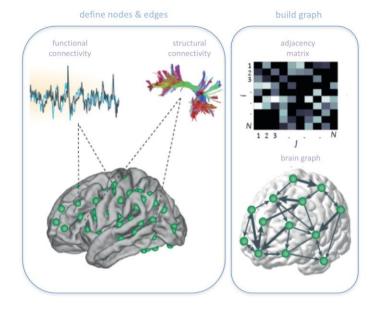
IPS 35
Hypothesis Testing on Neuroimaging Brain Networks
with Deep Generative Neural Networks

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19th August 2019, 2.00 pm – 3.40 pm

Outline/Content

- 1. Introduction: Brain networks
- 2. Problem Statement : high dimensionality & multiple comparisons
- 3. Generative Deep Neural Network (GDNN) Models
- 4. Model Formulation with GDNN
- 5. Hypothesis Testing with GDNN
- 6. Model Accuracy & Utility of Diverse GDNN Models
- 7. Challenges for Future Work

Brain Networks

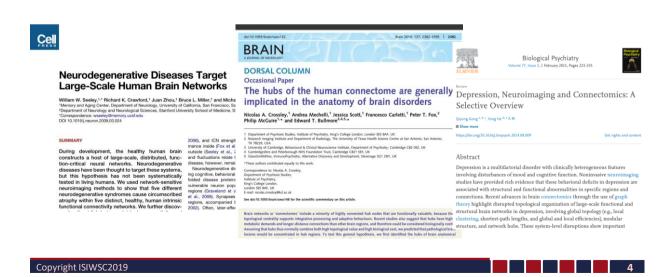


Slide from Alex Fornito, Monash Univ

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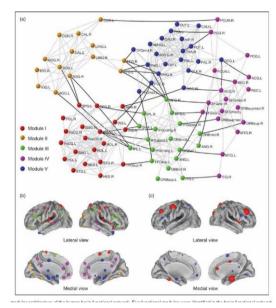
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Brain Networks, Degeneration & Disease



Problem Statement

- High dimensionality of graph edges
- Challenging to acquire sufficient brain datasets
- Fisherian hypothesis test leads to multiple comparisons and inflated false positives / negatives



YJ He, AC Evans, Curr Op Neurology 2010

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Statistical Model of Brain Networks

- Brain nodes represented by Joint probability of k random processes, $n_1, n_2, ..., n_k$ $P(\mathbf{n}) = P(n_1, n_2, ..., n_k)$
- Brain network are random variables, w_{ij} describing pairwise relationship between random processes which is fully described by joint distribution

$$P(\mathbf{w}) = P(w_{11}, w_{12}, ..., w_{1k}, w_{21}, w_{22}, ..., w_{2k}, ..., w_{kk})$$

Some examples of statistical pairwise relationship

$$w_{ij} = n_i * n_j$$

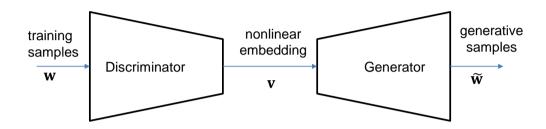
$$w_{ij} = \log P(n_i, n_j) - \log P(n_i) P(n_j)$$

- Correlation
- Mutual Entropy

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Generative Deep Neural Network Models

- Deep neural networks that learn a statistical model of the underlying dataset
- Purpose is to generate "novel" sample from that distribution

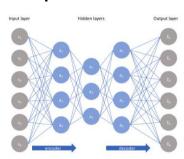


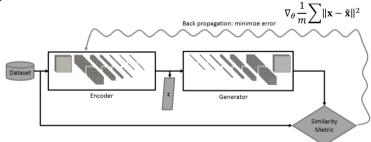
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Generative Deep Neural Network Models

Example 1: Autoencoders



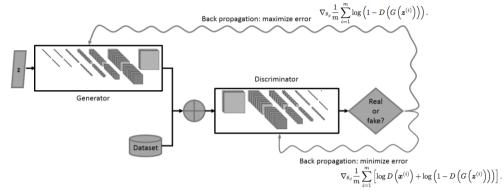


Generator = Discriminator

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Generative Deep Neural Network Models

Example 2: Generative Adversarial Networks



- Generator distinct from discriminator
- Learning is a competitive game between discriminator & generator optimization

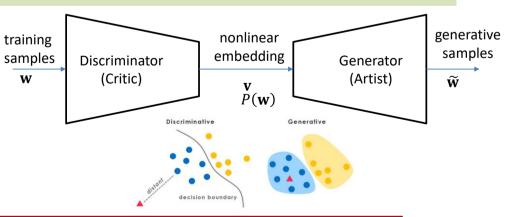
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Generative Deep Neural Network Models



Training a GDNN is a model formulation on the dataset



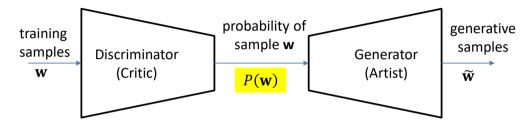
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Model Formulation with GDNN

Insight 2

Compare models, not data points to avoid multiple comparisons

- In general, the non linear embedding is a vector, \mathbf{v} with each element $\mathbf{v}_i \in [0,1]$
- Then the learnt probability distribution on the brain network dataset, $\tilde{P}(\mathbf{w}) = \prod_i \mathbf{v_i}$



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Benefits of GDNN Hypothesis Testing

- Hypothesis tested by comparing models of data rather than data points. Avoid false positives arising from random test in each dimension of data.
- With appropriate distance measure, test can rank degree of similarity
- Generative models allow deeper investigation into loci of differences not data constrained.

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Hypothesis Testing with GDNN

- $\tilde{P}(\mathbf{w})$ is a model on the brain network dataset, \mathbf{w} learnt by DGNN
- Hypothesis testing becomes a comparison between \tilde{P}_d (**w**) and a reference distribution $\tilde{P}_0(\mathbf{w})$ which is also learnt
 - brain networks from diseased cohort W. brain networks from healthy control cohort $H0: \mathbf{w}_d$ is not different from \mathbf{w}_0 Reject HO if sufficient difference in the

hypothesized probabilities

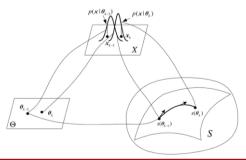
 $\tilde{P}_d(\mathbf{w}_d) \neq \tilde{P}_0(\mathbf{w}_0)$

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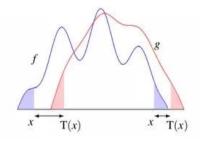
Hypothesis Testing with GDNN

- Accept or reject hypothesis based on magnitude of a distance metric
- What is a suitable distance metric between distributions? Two metrics known from Information Geometry & Optimal Transport Active research ongoing to unify both metrics

Fisher-Rao distance



Wasserstein / Earth Mover distance



Hypothesis Testing with GDNN

• Fisher-Rao distance, |dm|

Riemannian metric on statistical manifold given by $|d\mathbf{m}|^2 = \sum_i \sum_j g_{ij} d\theta_i d\theta_j$

where θ_i , θ_j are weights and biases in network 0 and network d respectively g_{ij} are elements in the **Fisher Information Matrix** defined by

$$g_{ij} = E\left[\frac{\partial \log \tilde{P}_0(\mathbf{w})}{\partial \theta_i} \frac{\partial \log \tilde{P}_d(\mathbf{w})}{\partial \theta_j}\right]$$

Choose direction of weights of each network with shortest geodesic distance between P_0 and P_d In matrix formulation

$$|d\mathbf{m}|^2 = \mathbf{\theta}_0^{\mathrm{T}} \mathbf{G} \mathbf{\theta}_d$$

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Generalizability of Fisher-Rao distance

- Invariance of the manifold (key result from Information Geometry)
- 1. Invariance to reparameterization

$$p_{\theta}(\mathbf{x}) = \bar{p}_{\xi}(\mathbf{x})$$
 and $\sum \theta^2 \neq \sum \xi^2$

2. Invariance under different representation

$$y = f(x)$$
 and $\int ||p_{\theta 1}(x) - p_{\theta 2}(x)||^2 dx \neq \int ||p_{\theta 1}(y) - p_{\theta 2}(y)||^2 dy$

- Implication: Fisher Rao distance is invariant if we change GDNN models (different number of weights, biases, layers, channels)
- Intuition: Distance metric between GDNN models defined on probability measures. Therefore parameterization of the probability measure should not matter

Accuracy of the GDNN model?

- If we are comparing GDNN models on data, how representative are the models on the underlying distribution of dataset?
- Bias-Variance-model complexity on the underlying dataset
- · Will diversity of models help?

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Utility of Diverse GDNN models

- Each GDNN model represents a point in the manifold of probability measures
- Multiple GDNN models on the same brain network dataset may be generated with
 - (i) different training instances
 - (ii) different learning algorithms
 - (iii) architectures
 - (iv) model complexity
- Any advantage for diversity of GDNN models on manifold of distributions?
 A Central Limit Theorem on Fréchet statistics?
 Hypothesize that multiple GDNN models cluster around the true distribution Clusters may be identified by Fréchet mean and variance.
- How to handle bias (underfit) and variance (overfit)?

Challenges for further work

- Choice of Fisher-Rao vs Wasserstein distance. Any benefits due to parameterization over GDNN parameters vs over brain variables
- Disentangling GDNN model bias-variance from true dataset distribution
- How degree of GDNN model diversity can ameliorate sample size requirement?
 Increase sample size improves the estimate of true probability distribution, can diversity help?
- Accuracy of comparisons at outliers/low probability events

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THANK YOU

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