Complex-time Representation of Longitudinal Processes & Topological Kime-Surface Analysis

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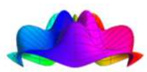
https://SOCR.umich.edu



STATISTICS ONLINE COMPUTATIONAL RESOURCE (SOCR) UNIVERSITY OF MICHIGAN   Slides online: Google *"SOCR News"*

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Outline

- ❑ *Kime representation of repeated measurement longitudinal processes*
  - ❑ Complex-time (*kime*) & rationale
  - ❑ Kime-phase, random sampling & Heisenberg's Uncertainty
  - ❑ Solutions of ultrahyperbolic wave equations
  - ❑ Mapping Time-series → Kime-surfaces
- ❑ *Kime-Phase Tomography (KPT), recovery of the phase distribution*
- ❑ *Applications: Spacekime Analytics*

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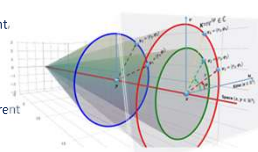
Complex-Time (Kime)

- ❑ At a given spatial location, $\boldsymbol{x}$, complex time (*kime*) is defined by $\kappa = re^{i\varphi} \in \mathbb{C}$, where:
  - ❑ The magnitude ($r > 0$) represents the longitudinal order of events characterizing the displacement in time, and
  - ❑ The event phase ($\varphi \sim \Phi(r)_{[-\pi;\pi)}$) is an angular displacement, event direction, reflecting a random sampling index
- ❑ There are multiple alternative parametrizations of kime in the complex plane
- ❑ Space-kime manifold is $\mathbb{R}^3 \times \mathbb{C}$:
  - ❑ $(\boldsymbol{x}, k_1)$ and $(\boldsymbol{x}, k_4)$ are spatially co-localized, but have different kime coordinates,
  - ❑ $(\boldsymbol{x}, k_1)$ and $(\boldsymbol{y}, k_1)$ are co-localized in kime, but represent different spatial locations,
  - ❑ $(\boldsymbol{x}, k_2)$ and $(\boldsymbol{x}, k_3)$ have the same spatial-locations and kime-directions, but appear ordered sequentially in time, $r_2 < r_1$.



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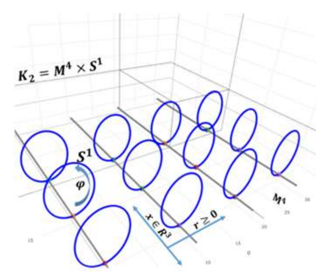
Historical Background: Kaluza-Klein Theory

- ❑ Theodor Kaluza (1921) developed a math extension of the classical general relativity theory to 5D. This included the metric, the field equations, the equations of motion, the stress-energy tensor, and the cylinder condition. Physicist Oskar Klein (1926) interpreted Kaluza's 3D+2D theory in quantum mechanical space and proposed that the fifth dimension was curled up and microscopic.
- ❑ The topology of the 5D Kaluza-Klein spacetime is $K_2 \cong M^4 \times S^1$, where M^4 is a 4D Minkowski spacetime and S^1 is a circle (non-traversable).



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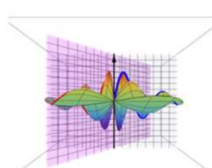
Rationale for Time ⟹ Kime Extension

Math – $Time$ is a special case of *kime*, $\kappa = |\kappa|e^{i\varphi}$ where $\varphi = 0$
Time ($\mathbb{R}^+$) is a subgroup of the multiplicative Reals group
Whereas kime ($\mathbb{C}$) is an algebraically *closed prime field* that naturally extends time
Time is ordered but *kime* is not!
Kime ($\mathbb{C}$) represents the smallest natural extension of time, as a complete field that agrees with time

Physics –
- The Problem of Time: Time has different meanings in *quantum mechanics* & *general relativity;* leading to a tension in formulating a *Quantum Gravity Theory* unifying the two … (DOI 10.1007/978-3-319-58848-3)
- (Base-field) $\mathbb{R}$ and $\mathbb{C}$ Hilbert-space quantum theories make different predictions (DOI: 10.1038/s41586-021-04160-4)

AI/Data Science – Random IID sampling, Bayesian reps, tensor modeling of $\mathbb{C}$ kimesurfaces, novel analytics



Wesson (2004, 2010)
Dinov & Velev (2021)
Wang et al. (2022)
Zhang et al. (2023)
Dinov & Shen (2024)

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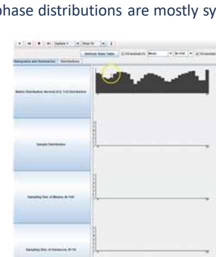
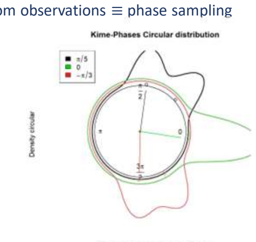
Random Sampling & Kime-Phase Paradigm

Kime phase distributions are mostly symmetric, random observations ≡ phase sampling



https://wiki.socr.umich.edu/index.php/SOCR_EduMaterials_Activities_GeneralCentralLimitTheorem



https://www.socr.umich.edu/TCIU/HTMLs/Chapter6_Kime_Phases_Circular.html

Dinov, Christou & Sanchez (2008)   Dinov & Velev (2021)

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Kime-Phase Measurement, Observability & Kime Operator

Kime-Phase Simulation – Repeated Spacetime Measurement

3 Processes – Green, Red and Blue colors (scatter points)

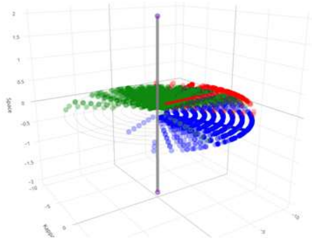
1 Fixed spatial location (vertical axis represents 1D space)

Repeated IID Measurements colocalized in 4D spacetime

3 Different Kime-Phase distributions (color-coded)

Radial displacement t = **time**

Angular (**phase**) location $\varphi \sim \Phi_{[-\pi,\pi)}(t)$



Wang et al., 2022 | Dinov & Velev (2021)

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Ultrahyperbolic PDEs: Wave Equation – Cauchy Initial Data

- For ultrahyperbolic PDEs, the initial value problem, determining the solution(s) for a given initial condition, is **ill-posed**, i.e., there's no guarantee of a global well-defined, stable, and unique solution!
- **Nonlocal constraints** yield the existence, uniqueness & stability of local and global solutions to the ultrahyperbolic wave equation under Cauchy initial data …

$$\underbrace{\sum_{i=1}^{d_s} \partial_{x_i}^2 u \equiv \Delta_x u(\boldsymbol{x},\boldsymbol{\kappa})}_{\text{spatial Laplacian}} = \underbrace{\Delta_{\boldsymbol{\kappa}} u(\boldsymbol{x},\boldsymbol{\kappa}) \equiv \sum_{i=1}^{d_t} \partial_{\kappa_i}^2 u}_{\text{temporal Laplacian}}, \quad \underbrace{\begin{cases} u_o = u\Big(\underbrace{\boldsymbol{x}}_{x\in D_s}, \underbrace{0,\boldsymbol{\kappa}_{-1}}_{\boldsymbol{\kappa}\in D_t}\Big) = f(\boldsymbol{x},\boldsymbol{\kappa}_{-1}) \\ u_1 = \partial_{\kappa_1} u(\boldsymbol{x},0,\boldsymbol{\kappa}_{-1}) = g(\boldsymbol{x},\boldsymbol{\kappa}_{-1}) \end{cases}}_{\text{initial conditions (Cauchy Data)}}$$

where $\boldsymbol{x} = (x_1, x_2, \ldots, x_{d_s}) \in \mathbb{R}^{d_s}$ and $\boldsymbol{\kappa} = (\kappa_1, \kappa_2, \ldots, \kappa_{d_t}) \in \mathbb{R}^{d_t}$ are the Cartesian coordinates in the d_s space and d_t time dims.

Stable local solution over a Fourier frequency region defined by nonlocal constraints $|\boldsymbol{\xi}| \geq |\boldsymbol{\eta}_{-1}|$:

$$\hat{u}\Big(\boldsymbol{\xi}, \underbrace{\kappa_1, \boldsymbol{\eta}_{-1}}_{\boldsymbol{\eta}}\Big) = \cos\Big(2\pi\,\kappa_1\sqrt{|\boldsymbol{\xi}|^2 - |\boldsymbol{\eta}_{-1}|^2}\Big)\underbrace{\hat{u}_o(\boldsymbol{\xi},\boldsymbol{\eta}_{-1})}_{c_1} + \sin\Big(2\pi\,\kappa_1\sqrt{|\boldsymbol{\xi}|^2 - |\boldsymbol{\eta}_{-1}|^2}\Big)\underbrace{\frac{\hat{u}_1(\boldsymbol{\xi},\boldsymbol{\eta}_{-1})}{2\pi\sqrt{|\boldsymbol{\xi}|^2 - |\boldsymbol{\eta}_{-1}|^2}}}_{c_2},$$

$$\text{where } \mathcal{F}\begin{pmatrix} u_o \\ u_1 \end{pmatrix} = \begin{pmatrix} \hat{u}_o \\ \hat{u}_1 \end{pmatrix} = \begin{pmatrix} \hat{u}_o(\boldsymbol{\xi},\boldsymbol{\eta}_{-1}) \\ \hat{u}_1(\boldsymbol{\xi},\boldsymbol{\eta}_{-1}) \end{pmatrix} = \begin{pmatrix} \hat{u}(\boldsymbol{\xi},\boldsymbol{\eta}_{-1}) \\ \partial_{\kappa_1}\hat{u}(\boldsymbol{\xi},\boldsymbol{\eta}_{-1}) \end{pmatrix}.$$

$$u\Big(\boldsymbol{x}, \underbrace{\kappa_1, \boldsymbol{\kappa}_{-1}}_{\boldsymbol{\kappa}}\Big) = \mathcal{F}^{-1}(\hat{u})(\boldsymbol{x},\boldsymbol{\kappa}) = \int_{\hat{D}_s \times \hat{D}_{t-1}} \hat{u}(\boldsymbol{\xi},\kappa_1,\boldsymbol{\eta}_{-1}) \times e^{2\pi i\langle x,\xi\rangle} \times e^{2\pi i\langle \kappa_{-1},\eta_{-1}\rangle} d\boldsymbol{\xi}\, d\boldsymbol{\eta}_{-1}\ .$$

Craig & Weinstein (2008) | Wang et al. (2022) | Dinov & Velev (2021)

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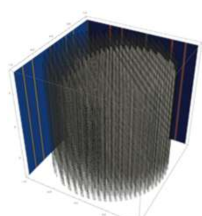
Ultrahyperbolic Wave Equation – Cauchy Initial Data

- Math Generalizations:

  Derived other spacekime concepts: law of addition of velocities, energy-momentum conservation law, stability conditions for particles moving in spacekime, conditions for nonzero rest particle mass, causal structure of spacekime, and solutions of the ultrahyperbolic wave equation under Cauchy initial data …



(Example Solution in 2D *space* + 2D *kime*)

Wang et al., 2022 | Dinov & Velev (2021)

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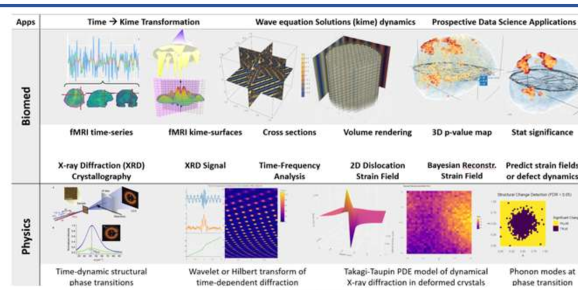
Idea: Longitudinal Data ⟹ Kime-Transforms ⟹ PDEs ⟹ AI



Rossbach, et al., 2019 | Wang, et al., 2022 | Dinov & Velev (2021)

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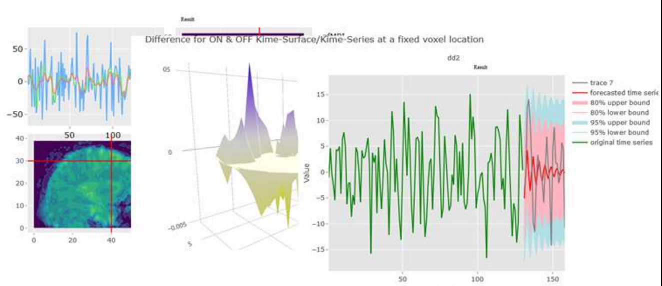
Spacetime Time-series ⟹ Spacekime Kimesurfaces ⟹ TLM



Zhang et al., 2022 | Dinov & Velev (2021)

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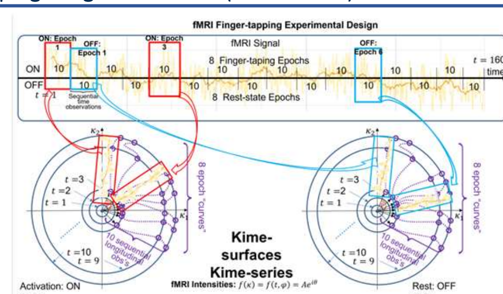
Mapping Longitudinal Data (Time-series) ⟹ Kime-surfaces



Zhang et al., 2022 | Dinov & Velev (2021)

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(Analytic) Mapping Time-series ⟹ Kime-surfaces

Apply the Inverse Laplace Transform, ILT ($\mathcal{L}^{-1}$) to reconstruct a time-series, $f(t) = \mathcal{L}^{-1}(F)(t)$:

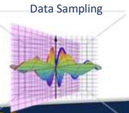
$$\boldsymbol{F(z)} = \mathcal{L}(f) = \underbrace{\frac{1}{z+1}}_{F_1(z)=\mathcal{L}(f_1(t)=e^{-t})} + \underbrace{\frac{1}{z^2+1}}_{F_2(z)=\mathcal{L}(f_2(t)=\sin(t))} \times \underbrace{\frac{z}{z^2+1}}_{F_3(z)=\mathcal{L}(f_3(t)=\cos(t))} + \underbrace{\frac{1}{z^2}}_{F_4(z)=\mathcal{L}(f_4(t)=t)} ,$$

$$\boldsymbol{f(t)} = \boldsymbol{\mathcal{L}^{-1}(F)} = \mathcal{L}^{-1}(F_1 + F_2 \times F_3 + F_4)) = \mathcal{L}^{-1}(F_1) + \Big(\underbrace{\mathcal{L}^{-1}(F_2) * \mathcal{L}^{-1}(F_3)}_{\text{convolution}}\Big)(t) + \mathcal{L}^{-1}(F_4) =$$

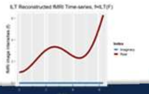
$$\mathcal{L}^{-1}\big(\mathcal{L}(f_1)\big)(t) + \Big(\mathcal{L}^{-1}\big(\mathcal{L}(f_2)\big) * \mathcal{L}^{-1}\big(\mathcal{L}(f_3)\big)\Big)(t) + \mathcal{L}^{-1}\big(\mathcal{L}(f_4)\big)(t) ,$$

$$\boldsymbol{f(t)} = \mathcal{L}^{-1}(F)(t) = f_1(t) + (f_2 * f_3)(t) + f_4(t) = e^{-t} + \int_0^t \sin(\tau) \times \cos(t-\tau)d\tau + t = \boldsymbol{t + e^{-t} + \frac{t\sin(t)}{2}} .$$

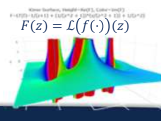
Repeated Longitudinal Data Sampling



$f(t) = \mathcal{L}^{-1}\big(\mathcal{L}(f)\big)(t)$

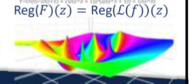


$F(z) = \mathcal{L}\big(f(\cdot)\big)(z)$



Inverse stereographic projection

$\mathrm{Reg}(F)(z) = \mathrm{Reg}(\mathcal{L}(f))(z)$



Shen et al., 2024 | Zhang et al., 2022 | Dinov & Velev (2021)

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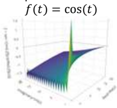
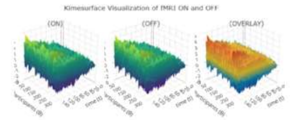
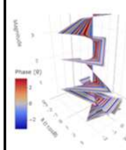
Mapping Longitudinal Data (Time-series) ⟹ Kime-surfaces

Laplace Transform
$f(t) = \cos(t)$

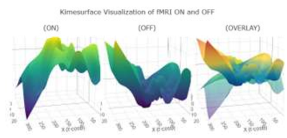




Raw simulated fMRI On/Off data
Low SNR; y axis is kime-phase, indexing the repeated runs within a participant and the multiple samples across participants



Multi-scale Wavelet
kime-surface reconstruction
f(t)=sin(2πt)+sin(2π3t)+ϵ



Simulated fMRI data, multiple runs
For *each* stimulus condition, each run/time point is Laplace-distributed, $\theta_{ij}(t_i) \sim Laplace(0, b_0 + \alpha\, t_i)$
Regularize the 2D (t, θ) ***domain*** by fitting a *thin-plate spline* (TPS)

https://www.socr.umich.edu/TCIU/HTMLs/Chapter6_TCIU_MappingLongitudinalTimeseries_2_Kimesurfaces.html

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Kime-Phase Tomography (KPT), phase recovery

- ❑ *Kime representation of repeated measurement longitudinal processes*
  - ❑ Complex-time (*kime*) & rationale
  - ❑ Kime-phase, random sampling & Heisenberg's Uncertainty
  - ❑ Solutions of ultrahyperbolic wave equations
  - ❑ Mapping Time-series → Kime-surfaces
- ❑ ***Kime-Phase Tomography (KPT), recovery of the phase distribution***
- ❑ *Spacekime Analytic Applications*

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Kime-Phase Tomography (KPT), phase recovery

Definition 1 (*Kime-Domain Signal Space*). Let $\mathcal{H}_t = L^2(\mathbb{R})$ be the Hilbert space of square-integrable complex-valued functions on the time domain, with inner product $\langle f, g\rangle_{\mathcal{H}_t} = \int_{\mathbb{R}} f(t)\overline{g(t)}\, dt$.

Definition 2 (*Phase-Domain Space*). Let $\mathcal{H}_\theta = L^2([-\pi, \pi])$ be the Hilbert space of square-integrable functions on the phase domain, with inner product $\langle \psi, \phi\rangle_{\mathcal{H}_\theta} = \int_{-\pi}^{\pi} \psi(\theta)\overline{\phi(\theta)}\, d\theta$ equipped with periodic boundary conditions $\psi(-\pi) = \psi(\pi)$.

Definition 3 (*Kime Space*). The kime space $\mathcal{K}$ is defined as the tensor product $\mathcal{H}_t \otimes \mathcal{H}_\theta$, representing signals in both time and phase domains.

Definition 4 (*Reproducing Kernel Hilbert Space, RKHS*). The RKHS $\mathcal{R}_K$ is a subspace of $\mathcal{H}_t$ with reproducing kernel $K: \mathbb{R} \times \mathbb{R} \to \mathbb{C}$ satisfying

- For any $t \in \mathbb{R}$, $K(\cdot, t) \in \mathcal{R}_K$, and
- For any $f \in \mathcal{R}_K$ and $t \in \mathbb{R}$, $f(t) = \langle f, K(\cdot, t)\rangle_{\mathcal{R}_K}$

Definition 5 (*Kime-Phase Distribution*). A kime-phase distribution $\Phi(\theta; t)$ is a time-dependent probability density function on $[-\pi, \pi]$ satisfying $\Phi(\theta; t) \geq 0$, $\int_{-\pi}^{\pi} \Phi(\theta; t)\, d\theta = 1 \quad \forall t \in \mathbb{R}$.

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Kime-Phase Tomography (KPT), phase recovery

Definition 6 (*Complex Kime*). For each time t, the complex kime is defined as $\kappa(t) = te^{i\theta(t)}$, where $\theta(t) \sim \Phi(\cdot; t)$.

Definition 7 (*Primary Kime Operators*). For a *signal* $s(t)$, the *primary kime operators* include

1. *Time-domain operator*: $K_1: \mathcal{H}_t \to \mathcal{H}_t$ defined by $K_1[s](t) = t \cdot s(t)$.
2. *Frequency-Domain Operator*: $K_2: \mathcal{H}_t \to \mathcal{H}_t$ defined by $K_2[s](t) = -i\frac{d}{dt} s(t)$, and
3. *Scale-Domain Operator*: For a mother wavelet $\psi \in \mathcal{H}_t$, let $W_\psi[s](a, b) = \langle s, \psi_{a,b}\rangle_{\mathcal{H}_t}$ be continuous wavelet transform with $\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right)$. Then, the scale-domain operator $K_3: \mathcal{H}_t \to \mathcal{H}_t$ is
$$K_3[s](t) = \int_{\mathbb{R}} \int_{\mathbb{R}_+} W_\psi[s](a, b) \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right) \frac{da\, db}{a^2}.$$
4. *Phase-Domain Operators*: In $\mathcal{H}_\theta$, we can define a pair of QM-equivalent phase-domain operators:
   a. *Position operator*: $\Theta[\phi](\theta) = \theta \cdot \phi(\theta)$, and
   b. *Momentum operator*: $P[\phi](\theta) = -i\frac{d}{d\theta}\phi(\theta)$.
5. *RKHS Projection Operator*: Given a kernel K, $\mathcal{P}_K: \mathcal{H}_t \to \mathcal{R}_K$ is defined by $\mathcal{P}_K[s](t) = \int_{\mathbb{R}} s(\tau)K(t, \tau)\, d\tau$.

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Kime-Phase Tomography (KPT), phase recovery

Definition 8 (Observable Signal). An observable kime-signal $s(t)$ with amplitude $A(t)$ and phase $\phi(t)$ is defined as $s(t) = A(t)e^{i\phi(t)}$, where $\phi(t)$ is sampled from distribution $\Phi_{[-\pi,\pi)}(\cdot; t)$.

Definition 9 (fMRI BOLD Signal Model). In fMRI, the observed BOLD signal $x(t)$ can be modeled as $x(t) = \int_{\mathbb{R}} h(t-\tau)s(\tau)\, d\tau + \epsilon(t)$, where $h(t)$ is the *hemodynamic response function* and $\epsilon(t)$ is *noise.*

This kime-operator framework is used for kime-phase recovery using repeated measurement observations of a controlled experiment, e.g., repeated fMRI runs in an event-related block design.

Theorem 1 (*Time-Frequency Commutation*). The operators K_1 (time-domain operator) and K_2 (frequency-domain operator) are *incompatible*, i.e., they have a non-trivial commutator, $[K_1, K_2] = K_1K_2 - K_2K_1 = i\mathcal{I}$, where $\mathcal{I}$ is the identity operator on $\mathcal{H}_t$. This indicates that the phase-reconstructions corresponding to this pair of kime-operators differentially probe the kime-phase and jointly, they recover complementary phase information.

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Kime-Phase Tomography (KPT), phase recovery

Theorem 2 (*Uncertainty Relation*). Given a signal $s \in \mathcal{H}_t$, time & frequency operators are non-commutative

$$\Delta K_1 \cdot \Delta K_2 \geq \frac{1}{2}|\langle [K_1, K_2] \rangle| = \frac{1}{2},$$

where $\Delta K_j = \sqrt{\langle K_j^2 \rangle - \langle K_j \rangle^2}$ for $j = 1,2$ and expectations are with respect to s.

Theorem 3 (*RKHS Representation*). Given a signal $s \in \mathcal{H}_t$ and a reproducing kernel K, the phase function $\phi(t)$ can be represented as the *complex argument* of the RKHS projection operator, $\mathcal{P}_K: \mathcal{H}_t \to \mathbb{C} \ni \mathcal{P}_K[s](t)$, i.e., $\underbrace{\phi(t) = \mathrm{Arg}(\mathcal{P}_K[s](t))}_{Phase\ Estimate}$.

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Kime-Phase Tomography (KPT), phase recovery

Lemma (*Phase Recovery from Multiple Bases*). Given repeated observations in multiple non-commuting bases defined by (time, freq & scale) operators K_1, K_2, K_3, the kime-phase distribution $\Phi(\theta; t)$ can be *uniquely determined* if the observations are sufficient.

Theorem 4 (*Generalized Phase Recovery from Multiple Bases*). Given a kime-phase distribution $\Phi(\theta; t)$ and assuming sufficient observations in multiple non-commuting bases defined by operators K_1, K_2, K_3, the phase distribution can be uniquely determined under certain uniqueness conditions

1. *Trigonometric Moment Identifiability*: A circular distribution is uniquely determined by its complete set of trigonometric moments $\{\alpha_k, \beta_k\}_{k=1}^{\infty}$ where $\alpha_k = \mathbb{E}[\cos(k\theta)]$ and $\beta_k = \mathbb{E}[\sin(k\theta)]$,
2. *Information Complementarity*: The (time, freq & scale) operators K_1, K_2, K_3 must provide complementary information about different moments of the phase distribution, and
3. *Sufficiency Condition*: The observations must constrain enough trigonometric moments to uniquely specify $\Phi(\theta; t)$ within the class of distributions being considered.

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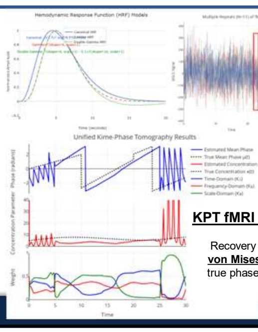
Algorithm: Kime-Phase Tomography (KPT), *fMRI Sim*

Input: BOLD time series $\{x_n(t)\}_{n=1}^{N}$ and kernel K
Output: Estimated phase $\hat{\phi}(t)$

1. Project each signal to RKHS: $s_n^K(t) = \mathcal{P}_K[s_n](t)$
2. Compute time-domain phase:
$$\phi_1^K(t) = \arg(t \cdot s_n^K(t))$$
3. Compute frequency-domain phase using STFT, $\phi_2^K(t) = \arg(\mathcal{F}[s_n^K \cdot w](t, \omega_{\max}))$, where $\omega_{\max}$ is the frequency with maximum power
4. Compute scale-domain phase using CWT: $\phi_3^K(t) = \arg\left(W_\psi[s_n^K](a_{\max}, t)\right)$, where $a_{\max}$ is the scale with maximum power
5. Compute weighted average (*ensemble KPT*):
$$\hat{\phi}(t) = \arg\left(\sum_{j=1}^{3} w_j(t) e^{i\phi_j^K(t)}\right).$$



KPT fMRI Simulation

Recovery of ϕ under **von Mises($\mu(t), \kappa(t)$)** true phase distribution

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Applications: Spacekime Analytics

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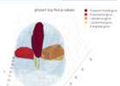
Example: Tensor-based Linear Modeling of fMRI

3-Step Analysis: registering the fMRI data into a brain atlas space, 56 ROIs, tensor linear modeling, post-hoc FDR processing & selection of large clusters of significant voxels are identified within the important ROIs:

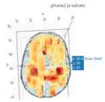
$$Y = \underbrace{\langle X, B \rangle}_{\text{tensor product}} + E$$

The dimensions of the time-tensor Y are $\overbrace{160}^{\text{time}} \times \overbrace{a \times b \times c}^{\text{ROI b-b}}$, where the tensor elements represent the response variable $Y[t, x, y, z]$, i.e., fMRI intensity. For fMRI magnitude (real-valued signal), the design kime-tensor X dimensions are:

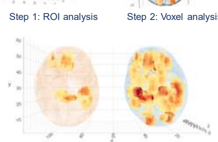
$$\underbrace{10 * 8}_{\text{Kime}(\text{Time}*e^{i \times Repeat})} \times \underbrace{State}_{\text{Stim vs. Rest (2)}} \times \underbrace{4}_{\text{effects}} \times \underbrace{1}_{\mathbb{R}}.$$



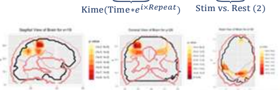
Step 1: ROI analysis



Step 2: Voxel analysis



Voxel-based TLM/Analysis Corrected (step 3, left) vs. Raw (step 2, right)



Step 3: 2D voxel analysis projections (finger-tapping task modeling)

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Spacekime Open-Problems

- ❑ There are many unsolved **abstract mathematical** challenges, e.g., space-kime ergodicity, metric tensor, kime-operator(s), etc.
- ❑ **Numerical & Computational** problems, e.g., reliable kime-phase tomography (KPT), optimal time-series ⟹ kime-surface reconstructions, etc.
- ❑ **Physics** parallels, e.g., contrasting QM vs. Spacekime predictions, physical observability, spacekime measurement, and kime-operator formalism
- ❑ **Analytical** challenges, e.g., new AI techniques for kime-surfaces, analytical verifiability & falsifiability of spacekime theory

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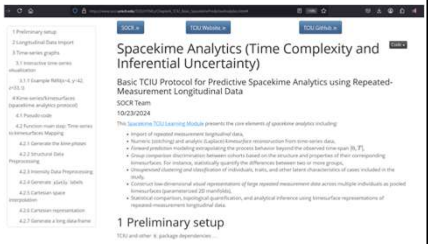
Spacekime Analytics Tutorial

TCIU/Spacekime Analytics Tutorial:

Basic TCIU Protocol for Predictive Spacekime Analytics using Longitudinal Data



https://www.socr.umich.edu/TCIU/HTMLs/Chapter6_TCIU_Basic_SpacekimePredictiveAnalytics.html

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Available Resources

- ❑ SOCR Motto – *"It's Online & Freely Accessible, Therefore it Exists!"*

- ❑ Pubs: https://socr.umich.edu/people/dinov/publications.html
- ❑ GitHub: https://github.com/SOCR
- ❑ Datasets: https://wiki.socr.umich.edu/index.php/SOCR_Data
- ❑ AI Apps: https://socr.umich.edu/HTML5/ (SOCR AI Bot)
- ❑ Demos: https://DSPA2.predictive.space
- ❑ Tutorials: https://TCIU.predictive.space & https://SpaceKime.org
- ❑ Website: https://socr.umich.edu
- ❑ Contact: statistics@umich.edu

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STATISTICS ONLINE COMPUTATIONAL RESOURCE (SOCR) https://SOCR.umich.edu

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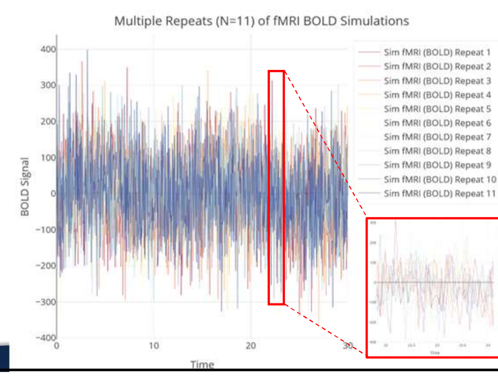

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