Quantum Mechanics Uncertainty, Data Science Inference & AI in Complex Time (*Kime*)

Ivo D. Dinov

Statistics Online Computational Resource
Michigan Center for Applied and Interdisciplinary Mathematics
Health Behavior & Biological Sciences
Computational Medicine & Bioinformatics
Michigan Institute for Data Science
University of Michigan

https://SOCR.umich.edu

Joint work with Milen V. Velev (BTU) & Yueyang Shen (UM)

Based on the book "*Data Science: Time Complexity, Inferential Uncertainty & Spacekime Analytics*"

STATISTICS ONLINE COMPUTATIONAL RESOURCE (SOCR) UNIVERSITY OF MICHIGAN

Slides Online: "SOCR News"

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Outline

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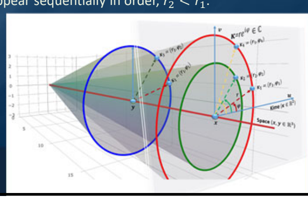
Complex-Time (*Kime*)

- ❑ At a given spatial location, $\boldsymbol{x}$, complex time (*kime*) is defined by $\kappa = re^{i\varphi} \in \mathbb{C}$, where:
  - ❑ the magnitude represents the longitudinal events order ($r > 0$) and characterizes the longitudinal displacement in time, and
  - ❑ event phase ($-\pi \le \varphi < \pi$) is an angular displacement, or event direction
- ❑ There are multiple alternative parametrizations of kime in the complex plane
- ❑ Space-kime manifold is $\mathbb{R}^3 \times \mathbb{C}$:
  - ❑ $(\boldsymbol{x}, k_1)$ and $(\boldsymbol{x}, k_4)$ have the same spacetime representation, but different spacekime coordinates,
  - ❑ $(\boldsymbol{x}, k_1)$ and $(\boldsymbol{y}, k_1)$ share the same kime, but represent different spatial locations,
  - ❑ $(\boldsymbol{x}, k_2)$ and $(\boldsymbol{x}, k_3)$ have the same spatial-locations and kime-directions, but appear sequentially in order, $r_2 < r_1$.



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Rationale for *Time* → *Kime* Extension

- ❑ **Math** – $Time$ is a special case of *kime*, $\kappa = |\kappa|e^{i\varphi}$ where $\varphi = 0$ (nil-phase)
  - algebraically a *multiplicative* (algebraic) group, (multiplicative) unity (identity) $= 1$
  - multiplicative inverses, multiplicative identity, associativity $t_1 * (t_2 * t_3) = (t_1 * t_2) * t_3$
  - The *time* domain ($\mathbb{R}^+$) is **not** a complete *algebraic field* $(+,*)$:
    - Additive unity (0), element additive inverse $(-t)$: $t + (-t) = 0$; is outside $\mathbb{R}^+$ (time-domain)
    - $x^2 + 1 = 0$ has no solutions in time (or in $\mathbb{R}$) ....

$$\text{Group}(*) \subseteq Ring\left(\underbrace{\overbrace{(+,*)}^{\text{Compatible operations}}}_{\text{associative \& distributive}}\right) \subseteq Field\ \overbrace{(+,*)}^{\text{Group}(+)}$$

  - Classical time ($\mathbb{R}^+$) is a *positive cone* over the field of the real numbers ($\mathbb{R}$)
  - Time forms a subgroup of the multiplicative group of the reals
  - Whereas kime ($\mathbb{C}$) is an algebraically *closed prime field* that naturally extends time
  - *Time* is ordered & *kime* is not – the kime magnitude preserves the intrinsic time order
  - Kime ($\mathbb{C}$) represents the smallest natural extension of time, complete filed that agrees with time
  - The *time* group is closed under addition, multiplication, and division (but not subtraction). It has the topology of $\mathbb{R}$ and the structure of a multiplicative topological group ≡ additive topological semigroup
- ❑ **Physics** –
  - ❑ Problem of time … (DOI 10.1007/978-3-319-58848-3)
  - ❑ $\mathbb{R}$ and $\mathbb{C}$ Hilbert-space quantum theories make different predictions (DOI: 10.1038/s41586-021-04160-4)
- ❑ **AI/Data Science** – Random IID sampling, Bayesian reps, tensor modeling of $\mathbb{C}$ kimesurfaces, novel analytics

Dinov & Velev (2021)

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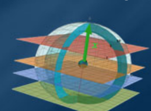
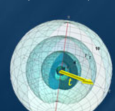
Uncertainty in 5D Spacekime

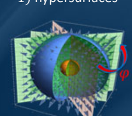
- ❑ 5D Space-Time-Matter Consortium showed that in a 5D universe with an extra time dimension, particle spacetime motion may be slightly modified by an extra force to produce a correlation between the momentum and position similar to the uncertainty relation in quantum mechanics
- ❑ One component of this additional force is parallel to the 4-velocity and explains the intrinsic Heisenberg uncertainty relation in the lower 4D spacetime embedding
- ❑ We can represent classical 4D spacetime Heisenberg uncertainty as a reduction of Einstein-like 5D deterministic dynamics
- ❑ (Paul Wesson) "*Heisenberg was right in 4D, because Einstein was right in 5D*"
- ❑ D-dimensional "generating" space *foliated* by a family of $(D-1)$ hypersurfaces



Space (x) Foliation of Spacekime



(Radial, t) Time-Foliation of Spacekime



(Angular, φ) Phase-Foliation of *Kime*

Wesson (2004, 2010) | Wesson & Overduin (2018) | Dinov & Velev (2021)

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Uncertainty in 5D Spacekime

In 5D space-time, $\Omega = \left(\frac{l}{L}\right)^2$ is the conformal factor, and L is a constant length defined in terms of the cosmological constant $\Lambda = -\epsilon\frac{3}{L^2}$, where, in the metric signature $(+,-,-,-)$, $\Lambda > 0$ for a spacelike extra coordinate and $\Lambda < 0$ for a timelike extra 5th coordinate, x^μ is the $(D-1)$ spacetime location, and l is the extra kime dimension.

The *canonical spacekime metric* is:

$$dS^2 = \frac{l^2}{L^2}\Sigma_0^{D-2}\Sigma_0^{D-2} g_{\alpha\beta}(x^\mu, l)dx^\alpha dx^\beta + \epsilon dl^2$$

The 4D components of the spacekime equations of motion can be written explicitly in terms of the fifth force f^μ measured in units of inertia mass, i.e., assuming $m = 1$:

$$\frac{du^\mu}{ds} + \sum_0^3\sum_0^3 \Gamma^\mu_{\beta\gamma}\, u^\beta\, u^\gamma = f^\mu, \qquad f^\mu \equiv \sum_{\alpha=0}^3\sum_{\beta=0}^3\left(-g^{\mu\alpha} + \frac{1}{2}u^\mu u^\alpha\right)\frac{dl}{ds}\frac{dx^\beta}{ds}\frac{\partial g_{\alpha\beta}}{\partial l}$$

The 5D component of the spacekime equation of motion is:

$$\frac{d^2l}{ds^2} - \frac{2}{l}\left(\frac{dl}{ds}\right)^2 - \frac{l}{L^2} = \frac{1}{2}\left[\frac{l^2}{L^2} + \left(\frac{dl}{ds}\right)^2\right]\Sigma_{\alpha=0}^3\Sigma_{\beta=0}^3 u^\alpha u^\beta \frac{\partial g_{\alpha\beta}}{\partial l}$$

In 5D spacekime, geodesic motion is perturbed by an extra force $f^\mu = f_\perp^\mu + f_\parallel^\mu$, where

- $f_\perp^\mu$ is normal to the 4-velocity u_μ, similar to other conventional forces, and $f_\perp^\mu u_\mu = 0$
- $f_\parallel^\mu$ is parallel to the 4-velocity u_μ, has no analog in 4D spacetime, and $f_\parallel^\mu u_\mu \neq 0$.

Wesson (2004, 2010) | Wesson & Overduin (2018) | Dinov & Velev (2021)

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Uncertainty in 5D Spacekime

- Assuming $m = 1, c = 1$, near the foliation leaf membrane hypersurface, we have
$$\langle dp \mid dx \rangle = \sum_{\mu=0}^{3} dp^{\mu}\, dx_{\mu} = L\left(\frac{dl}{l-l_0}\right)^2 = \frac{h}{m\,c}\left(\frac{dl}{l-l_0}\right)^2 \sim h$$
- This relation is derived from 5D Einstein deterministic dynamics and is analogous to the quantum mechanics uncertainty principle in 4D Minkowski spacetime
- In spacetime, Heisenberg's uncertainty appears because of lack of sufficient information about the second kime dimension, l.
- In Minkowski 4D spacetime, the lack of kime-phase information naturally leaves one degree of freedom in the system, which appears as Heisenberg's uncertainty.
- **In Statistics, Data Science, ML/AI, and longitudinal analytics, this extra degree of freedom is represented as process stochasticity -- random sampling from an underlying prob-distribution**
- Spacekime formulation of the 4D spacetime observation of the Heisenberg's principle also supports the de Broglie-Bohm theory, which provides an explicit deterministic model of a system configuration and its corresponding wavefunction
- 4D probabilistic spacetime is a spacekime embedding with an added degrees of freedom
- Bell's theorem suggests that any deterministic hidden-variable theory, which is consistent with quantum mechanics predictions, has to be non-local. This implies the existence of instantaneous, faster than the speed of light, interactions between particles that are significantly separated in 3D space (non-local relations).

Wesson (2004, 2010) | Bell (1964) | Dinov & Velev (2021)

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Ultrahyperbolic Wave Equation – Cauchy Initial Data

- **Nonlocal constraints** yield the existence, uniqueness & stability of local and global solutions to the ultrahyperbolic wave equation under Cauchy initial data …

$$\underbrace{\sum_{i=1}^{d_s} \partial_{x_i}^2 u \equiv \Delta_{\boldsymbol{x}} u(\boldsymbol{x},\boldsymbol{\kappa})}_{\text{spatial Laplacian}} = \underbrace{\Delta_{\boldsymbol{\kappa}} u(\boldsymbol{x},\boldsymbol{\kappa}) \equiv \sum_{i=1}^{d_t} \partial_{\kappa_i}^2 u}_{\text{temporal Laplacian}}, \quad \underbrace{\left| \begin{array}{l} u_o = u\Big(\underbrace{\boldsymbol{x}}_{\boldsymbol{x}\in D_s}, \underbrace{0,\boldsymbol{\kappa}_{-1}}_{\boldsymbol{\kappa}\in D_t}\Big) = f(\boldsymbol{x},\boldsymbol{\kappa}_{-1}) \\ u_1 = \partial_{\kappa_1} u(\boldsymbol{x},0,\boldsymbol{\kappa}_{-1}) = g(\boldsymbol{x},\boldsymbol{\kappa}_{-1}) \end{array} \right.}_{\text{initial conditions (Cauchy Data)}}$$

where $\boldsymbol{x} = (x_1, x_2, \dots, x_{d_s}) \in \mathbb{R}^{d_s}$ and $\boldsymbol{\kappa} = (\kappa_1, \kappa_2, \dots, \kappa_{d_t}) \in \mathbb{R}^{d_t}$ are the Cartesian coordinates in the d_s space and d_t time dims.

Stable local solution over a Fourier frequency region defined by nonlocal constraints $|\boldsymbol{\xi}| \geq |\boldsymbol{\eta}_{-1}|$:

$$\hat{u}\Big(\boldsymbol{\xi}, \underbrace{\kappa_1, \boldsymbol{\eta}_{-1}}_{\boldsymbol{\eta}}\Big) = \cos\left(2\pi\,\kappa_1\sqrt{|\boldsymbol{\xi}|^2 - |\boldsymbol{\eta}_{-1}|^2}\right) \underbrace{\hat{u}_o(\boldsymbol{\xi},\boldsymbol{\eta}_{-1})}_{c_1} + \sin\left(2\pi\,\kappa_1\sqrt{|\boldsymbol{\xi}|^2 - |\boldsymbol{\eta}_{-1}|^2}\right) \underbrace{\frac{\hat{u}_1(\boldsymbol{\xi},\boldsymbol{\eta}_{-1})}{2\pi\sqrt{|\boldsymbol{\xi}|^2 - |\boldsymbol{\eta}_{-1}|^2}}}_{c_2},$$

where $\mathcal{F}\begin{pmatrix} u_o \\ u_1 \end{pmatrix} = \begin{pmatrix} \hat{u}_o \\ \hat{u}_1 \end{pmatrix} = \begin{pmatrix} \hat{u}_o(\boldsymbol{\xi},\boldsymbol{\eta}_{-1}) \\ \hat{u}_1(\boldsymbol{\xi},\boldsymbol{\eta}_{-1}) \end{pmatrix} = \begin{pmatrix} \hat{u}(\boldsymbol{\xi},\boldsymbol{\eta}_{-1}) \\ \partial_{\kappa_1}\hat{u}(\boldsymbol{\xi},\boldsymbol{\eta}_{-1}) \end{pmatrix}$.

$$u\Big(\boldsymbol{x}, \underbrace{\kappa_1, \boldsymbol{\kappa}_{-1}}_{\boldsymbol{\kappa}}\Big) = \mathcal{F}^{-1}(\hat{u})(\boldsymbol{x},\boldsymbol{\kappa}) = \int_{\widehat{D}_s \times \widehat{D}_{t_{-1}}} \hat{u}(\boldsymbol{\xi},\kappa_1,\boldsymbol{\eta}_{-1}) \times e^{2\pi i \langle \boldsymbol{x},\boldsymbol{\xi}\rangle} \times e^{2\pi i \langle \boldsymbol{\kappa}_{-1},\boldsymbol{\eta}_{-1}\rangle} d\boldsymbol{\xi}\, d\boldsymbol{\eta}_{-1}.$$

Wang et al., 2022 | Dinov & Velev (2021)

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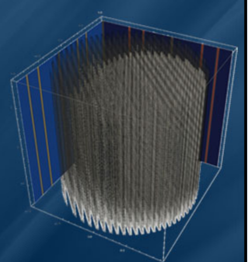
A *Spacekime* Solution to Wave Equation

- Math Generalizations: Derived other spacekime concepts: law of addition of velocities, energy-momentum conservation law, stability conditions for particles moving in spacekime, conditions for nonzero rest particle mass, causal structure of spacekime, and solutions of the ultrahyperbolic wave equation under Cauchy initial data …



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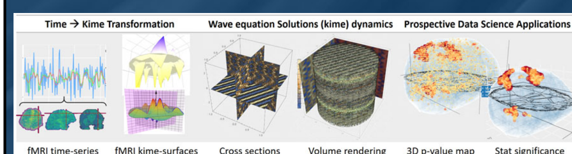
Kime transforms → PDEs → AI



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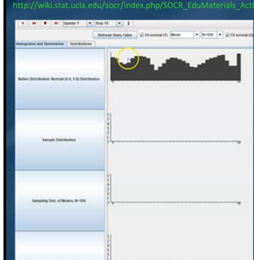
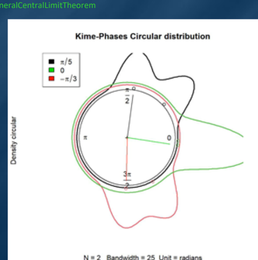
Random Sampling & Kime-Phase Paradigm

- Kime phase distributions are mostly symmetric, random observations ≡ phase sampling

http://wiki.stat.ucla.edu/socr/index.php/SOCR_EduMaterials_Activities_GeneralCentralLimitTheorem





https://www.socr.umich.edu/TCIU/HTMLs/Chapter6_Kime_Phases_Circular.html

Dinov, Christou & Sanchez (2008) Dinov & Velev (2021)

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(Many) Spacekime Open Math Problems

- **Ergodicity**

Let's look at particle velocities in the 4D Minkowski spacetime (X), a measure space where gas particles move spatially and evolve longitudinally in time. Let $\mu = \mu_{\boldsymbol{x}}$ be a measure on X, $f(\boldsymbol{x},t) \in L^1(X,\mu)$ be an integrable function (e.g., velocity of a particle), and $T: X \to X$ be a measure-preserving transformation at position $\boldsymbol{x} \in \mathbb{R}^3$ and time $t \in \mathbb{R}^+$.

A pointwise ergodic theorem argues that in a measure theoretic sense, the average of f (e.g., velocity) over all particles in the gas system at a fixed time, $\bar{f} = E_t(f) = \int_{\mathbb{R}^3} f(\boldsymbol{x},t)\,d\mu_{\boldsymbol{x}}$, will be equal to the average f of just one particle ($\boldsymbol{x}$) over the entire time span,

$$\tilde{f} = \lim_{n\to\infty}\left(\frac{1}{n}\sum_{m=0}^{n} f(T^m\boldsymbol{x})\right), \text{ i.e., (show) } \bar{f} \equiv \tilde{f}.$$

The spatial probability measure is denoted by $\mu_{\boldsymbol{x}}$ and the transformation $T^m\boldsymbol{x}$ represents the dynamics (time evolution) of the particle starting with an initial spatial location $T^o\boldsymbol{x} = \boldsymbol{x}$.

Investigate the ergodic properties of various transformations in the 5D **spacekime**:

$$\underbrace{\bar{f} \equiv E_{\kappa}(f) = \frac{1}{\mu_{\boldsymbol{x}}(X)} \int f\Big(\boldsymbol{x}, \underbrace{t,\phi}_{\kappa}\Big)\, d\mu_{\boldsymbol{x}}}_{\text{space averaging}} \overset{?}{=} \underbrace{\lim_{t\to\infty}\left(\frac{1}{t}\sum_{m=0}^{t}\left(\int_{-\pi}^{+\pi} f(T^m\boldsymbol{x},t,\phi)\,d\Phi\right)\right) \equiv \tilde{f}}_{\text{kime averaging}}$$

Dinov & Velev (2021)

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Mathematical-Physics ⟹ Data Science & AI

Physics	Data/Neuro Sciences
A **particle** is a small localized object that permits observations and characterization of its physical or chemical properties	An **object** is something that exists by itself, actually or potentially, concretely or abstractly, physically or incorporeal (e.g., person, subject, etc.)
An **observable** a dynamic variable about particles that can be measured	A **feature** is a dynamic variable or an attribute about an object that can be measured
Particle **state** is an observable particle characteristic (e.g., position, momentum)	**Datum** is an observed quantitative or qualitative value, an instantiation, of a feature
Particle **system** is a collection of independent particles and observable characteristics, in a closed system	**Problem**, aka Data System, is a collection of independent objects and features, without necessarily being associated with a priori hypotheses
Wave-function	**Inference-function**
Reference-Frame **transforms** (e.g., Lorentz)	Data **transformations** (e.g., wrangling, log-transform)
State of a system is an observed measurement of all particles ~ wavefunction	**Dataset (data)** is an observed instance of a set of datum elements about the problem system, $O = \{X, Y\}$
A **particle system is computable** if (1) the entire system is logical, consistent, complete and (2) the unknown internal states of the system don't influence the computation (wavefunction, intervals, probabilities, etc.)	**Computable data object** is a very special representation of a dataset which allows direct application of computational processing, modeling, analytics, or inference based on the observed dataset
…	…

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Spacekime Analytics: fMRI Example

- 3D Isosurface Reconstruction of (2D *space* × 1D *time*) fMRI signal

Spacetime Reconstruction using trivial phase-angle; kime=time=(magnitude, 0)	Spacekime Reconstruction using correct kime=(magnitude, phase)

3D pseudo-spacetime reconstruction:

$$f = \hat{h}(\underbrace{x_1, x_2}_{space}, \underbrace{t}_{time})$$

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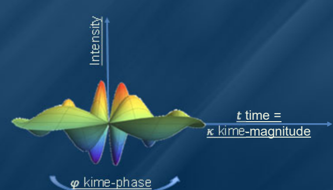
Spacekime Analytics: Kime-series = Surfaces (not curves)

In the 5D spacekime manifold, time-series curves extend to kime-series, i.e., surfaces parameterized by kime-magnitude (t) and the kime-phase (φ)

Kime-phase aggregating operators that can be used to transform standard time-series curves to spacekime kime-surfaces, which can be modeled, interpreted, and predicted using advanced spacekime analytics



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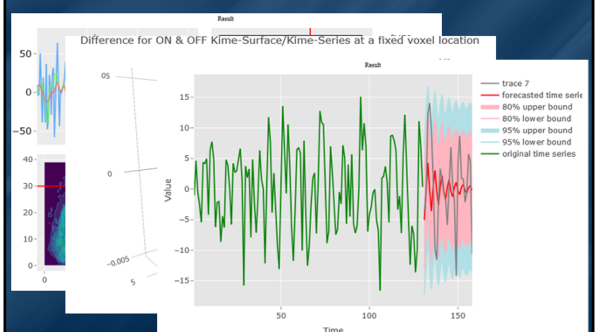
Spacetime Time-series ⟹ Spacekime Kimesurfaces ⟹ TLM



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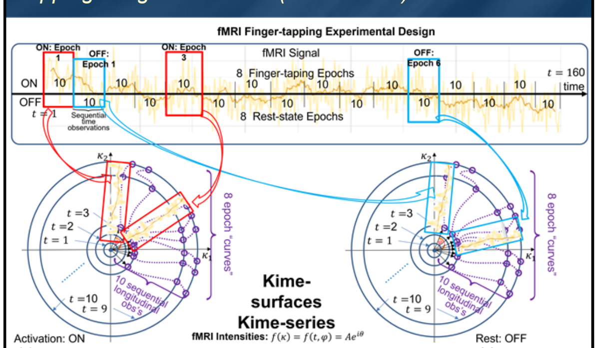
Mapping Longitudinal Data (Time-series) to Kime-Surfaces



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Mapping Longitudinal Data (Time-series) to Kime-Surfaces

The *forward* and *inverse* (continuous) Laplace transforms are defined below.

- For a given function (of time) $f(t): \mathbb{R}^+ \to \mathbb{C}$, the **Laplace transform** is the function of a complex frequency argument, $F(z) = \mathcal{L}(f)(z): \mathbb{C} \to \mathbb{C}$:

$$\mathcal{L}(f)(z) = F(z) = \int_0^\infty f(t)e^{-zt}\,dt\,.$$

- For a given function function of a complex frequency argument, $F(z)$, the **Inverse Laplace transform** (ILT) is the function of a positive real (time-like) argument $f(t) = \mathcal{L}^{-1}(F)(t): \mathbb{R}^+ \to \mathbb{C}$, which is defined in terms of a complex path integral (a.k.a. Bromwich integral or Fourier–Mellin integral):

$$f(t) = \mathcal{L}^{-1}(F)(t) = \frac{1}{2\pi i}\lim_{T\to\infty}\int_{\gamma-iT}^{\gamma+iT} e^{zt}F(z)dz\,,$$

where the parameter $\gamma \in \mathbb{R}$ is chosen so that the entire complex contour path of the integral is inside of the region of convergence of $F(z)$.

The Laplace transform plays an interesting role as an expected value in the field of probability and statistics. If X is a random variable, then its Laplace transform, i.e., the LT of its probability density function f_X, is given by the expectation of an exponential: $\mathcal{L}(X) = \mathcal{L}(f)(z) = E(e^{-zX})$.

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Mapping Longitudinal Data (Time-series) to Kime-Surfaces

Apply the ILT ($\mathcal{L}^{-1}$) to reconstruct a time-series, $\hat{f}(t) = \mathcal{L}^{-1}(F)(t)$,

$$F(z) = \mathcal{L}(f) = \underbrace{\frac{1}{z+1}}_{F_1(z)=\mathcal{L}(f_1(t)=e^{-t})} + \underbrace{\frac{1}{z^2+1}}_{F_2(z)=\mathcal{L}(f_2(t)=\sin(t))} \times \underbrace{\frac{z}{z^2+1}}_{F_3(z)=\mathcal{L}(f_3(t)=\cos(t))} + \underbrace{\frac{1}{z^2}}_{F_4(z)=\mathcal{L}(f_4(t)=t)},$$

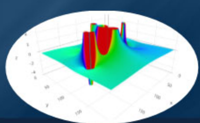
$$f(t) = \mathcal{L}^{-1}(F) = \mathcal{L}^{-1}(F_1 + F_2 \times F_3 + F_4)) =$$

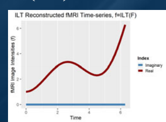
$$\mathcal{L}^{-1}(F_1) + \Big(\underbrace{\mathcal{L}^{-1}(F_2) * \mathcal{L}^{-1}(F_3)}_{\text{convolution}}\Big)(t) + \mathcal{L}^{-1}(F_4) =$$

$$\mathcal{L}^{-1}(\mathcal{L}(f_1))(t) + \Big(\mathcal{L}^{-1}(\mathcal{L}(f_2)) * \mathcal{L}^{-1}(\mathcal{L}(f_3))\Big)(t) + \mathcal{L}^{-1}(\mathcal{L}(f_4))(t),$$

$$f(t) = \mathcal{L}^{-1}(F)(t) = f_1(t) + (f_2 * f_3)(t) + f_4(t) =$$

$$e^{-t} + \int_0^t \sin(\tau) \times \cos(t-\tau) d\tau + t = t + e^{-t} + \frac{t\sin(t)}{2}.$$





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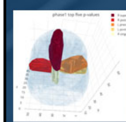
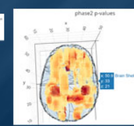
Tensor-based Linear Modeling of fMRI

3-Step Analysis: registering the fMRI data into a brain atlas space, 56 ROIs, tensor linear modeling, post-hoc FDR processing & selection of large clusters of significant voxels are identified within the important ROIs: $Y = \underbrace{\langle X, B \rangle}_{\text{tensor product}} + E$.

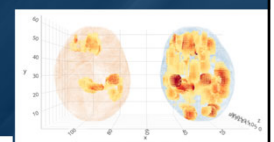
The dimensions of the time-tensor Y are $\overbrace{160}^{\text{time}} \times \overbrace{a \times b \times c}^{\text{ROI b-box}}$, where the tensor elements represent the response variable $Y[t, x, y, z]$, i.e., fMRI intensity. For fMRI magnitude (real-valued signal), the design kime-tensor X dimensions are: $\underbrace{10 * 8}_{\text{Kime}(\text{Time} * e^{i \times Repeat})} \times \underbrace{State}_{\text{Stim vs. Rest (2)}} \times \underbrace{4}_{\text{effects}} \times \underbrace{1}_{\mathbb{R}}$.



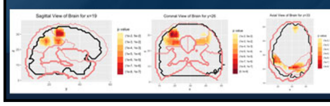
Step 1: ROI analysis



Step 2: Voxel analysis



Voxel-based TLM/Analysis
Corrected (step 3, left) vs. Raw (step 2, right)



Step 3: 2D voxel analysis projections (finger-tapping task modeling)

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Bayesian Inference Representation

- Suppose we have a single spacetime observation $X = \{x_{i_o}\} \sim p(x \mid \gamma)$ and $\gamma \sim p(\gamma \mid \varphi = \text{phase})$ is a process parameter (or vector) that we are trying to estimate.
- Spacekime analytics aims to make appropriate inference about the process X.
- The sampling distribution, $p(x \mid \gamma)$, is the distribution of the observed data X conditional on the parameter γ and the prior distribution, $p(\gamma \mid \varphi)$, of the parameter γ before the data X is observed, $\varphi = \text{phase aggregator}$.
- Assume that the hyperparameter (vector) φ, which represents the kime-phase estimates for the process, can be estimated by $\hat{\varphi} = \varphi'$.
- Such estimates may be obtained from an oracle (model distribution), approximated using similar datasets, acquired as phases from samples of analogous processes, derived via some phase-aggregation strategy, or computed via Laplace transform.
- Let the posterior distribution of the parameter γ given the observed data $X = \{x_{i_o}\}$ be $p(\gamma | X, \varphi')$ and the process parameter distribution of the kime-phase hyperparameter vector φ be $\gamma \sim p(\gamma \mid \varphi)$.

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Bayesian Inference Simulation

- Simulation example using 2 random samples drawn from mixture distributions each of $n_A = n_B = 10K$ observations:
  - $\{X_{A,i}\}_{i=1}^{n_A}$, where $X_{A,i} = 0.3U_i + 0.7V_i$, $U_i \sim N(0,1)$ and $V_i \sim N(5,3)$, and
  - $\{X_{B,i}\}_{i=1}^{n_B}$, where $X_{B,i} = 0.4P_i + 0.6Q_i$, $P_i \sim N(20,20)$ and $Q_i \sim N(100,30)$.
- The intensities of cohorts A and B are independent and follow different mixture distributions. We'll split the first cohort (A) into training (C) and testing (D) subgroups, and then:
  - Transform all four cohorts into Fourier k-space,
  - Iteratively randomly sample single observations from the (training) cohort C,
  - Reconstruct the data into spacetime using a single kime-magnitude value and alternative kime-phase estimates derived from cohorts B, C, and D, and
  - Compute the classical spacetime-derived population characteristics of cohort A and compare them to their spacekime counterparts obtained using a single C kime-magnitude paired with B, C, or D kime-phases.

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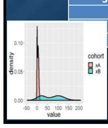
Bayesian Inference Simulation

Summary statistics for the original process (cohort A) and the corresponding values of their counterparts computed using the spacekime reconstructed signals based on kime-phases of cohorts B, C, and D. The estimates for the latter three cohorts correspond to reconstructions using a single spacetime observation (i.e., single kime-magnitude) and alternative kime-phase priors (in this case, kime-phases derived from cohorts B, C, and D).

	Spacetime	Spacekime Reconstructions (single kime-magnitude)		
Summaries	**(A) Original**	(B) Phase=Diff. Process	(C) Phase=True	(D) Phase=Independent
Min	-2.38798	-3.798440	-2.98116	-2.69808
1st Quartile	-0.89359	-0.636799	-0.76765	-0.76453
Median	0.03311	0.009279	-0.05982	-0.08329
Mean	**0.00000**	**0.000000**	**0.00000**	**0.00000**
3rd Quartile	0.75772	0.645119	0.72795	0.69889
Max	3.61346	3.986702	3.64800	3.22987
Skewness	0.348269	0.001021943	0.2372526	0.31398
Kurtosis	**-0.68176**	**0.2149918**	**-0.4452207**	**-0.3270084**



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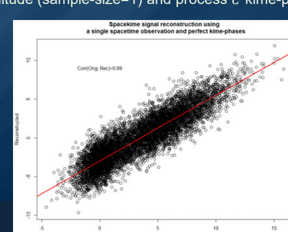
Bayesian Inference Simulation

The correlation between the original data (A) and its reconstruction using a single kime magnitude and the correct kime-phases (C) is $\rho(A, C) = 0.89$.

This strong correlation suggests that a substantial part of the A process energy can be recovered using only a single observation. In this case, to reconstruct the signal back into spacetime and compute the corresponding correlation, we used a single kime-magnitude (sample-size=1) and process C kime-phases.



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Bayesian Inference Simulation

Let's demonstrate the Bayesian inference corresponding to this spacekime data analytic problem using a simulated bimodal experiment:

$X_A = 0.3U + 0.7V$, where $U \sim N(0,1)$ and $V \sim N(5,3)$

Specifically, we will illustrate the Bayesian inference using repeated single spacetime observations from cohort A, $X = \{x_{i_o}\}$, and varying kime-phase priors ($\theta = \text{phase aggregator}$) obtained from cohorts B, C, or D, using different posterior predictive distributions.

Relations between the empirical data distribution (dark blue) and samples from the posterior predictive distribution, representing Bayesian simulated spacekime reconstructions (light-blue). The derived Bayesian estimates do not perfectly match the empirical distribution of the simulated data, yet there is clearly information encoding that is captured by the spacekime data reconstructions.

This signal compression can be exploited by subsequent model-based or model-free data analytic strategies for retrospective prediction, prospective forecasting, ML classification, AI derived clustering, and other spacekime inference methods.

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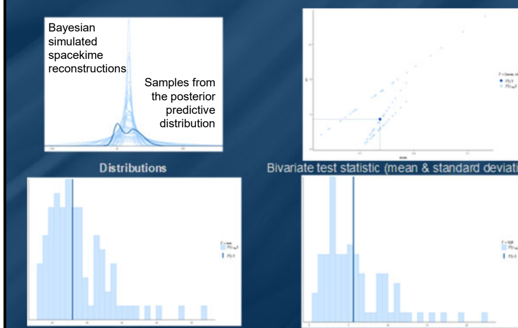
Bayesian Inference Simulation



Relations between the empirical data distribution (dark blue) and samples from the posterior predictive distribution, Bayesian simulated spacekime reconstructions (light-blue).

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Spacekime Analytics: Resources & Demos

- Tutorials
  - https://TCIU.predictive.space
  - https://SpaceKime.org
- R Package
  - https://cran.rstudio.com/web/packages/TCIU
- GitHub
  - https://github.com/SOCR/TCIU
- Pubs
  - https://socr.umich.edu/people/dinov/publications.html

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Acknowledgments

Slides Online: "SOCR News"

Funding

NIH: UL1TR002240, R01CA233487, R01MH121079, R01MH126137, T32GM141746
NSF: 1916425, 1734853, 1636840, 1416953, 0716055, 1023115

Collaborators

- **SOCR: Milen Velev, Yueyang Shen, Daxuan Deng, Zijing Li, Yongkai Qiu, Zhe Yin, Yufei Yang, Yuxin Wang, Rongqian Zhang, Yuyao Liu, Yupeng Zhang, Yunjie Guo, Simeone Marino**
- **UMich DCMB/MIDAS/MCAIM Centers**: Josh Welch, Maryam Bagherian, Lydia Bieri, Kayvan Najarian, Chris Monk, Issam El Naqa, HV Jagadish, Brian Athey

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